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ABSTRACT

Historically minorities, especially African-Americans, have been underrepresented in the mathematics and science fields. In order to attract more minorities as mathematics majors, these students must first experience success in the pre-calculus sequence and enroll in higher-level mathematics courses. The Georgia State University (GSU) Atlanta Metropolitan College (AMC) Partnership Project is a collaborative effort to increase articulation between developmental (non-credit) and regular (credit) mathematics courses within and between institutions. This project was created in order to facilitate transfer of students to GSU from AMC, which has a large number of students who transfer to GSU and a predominantly African-American student body. Content analysis was performed on all sections of the target courses, faculty and student surveys were administered, and routine classroom observations were made. The instructional practices employed at both institutions were found to be similar. Differences between the institutions that may affect transfer students include: (1) sequencing of instruction; (2) levels of classroom interaction; (3) and breadth of topic coverage. It was recommended that students transfer at the end of a sequence rather than in the middle. Appendices include: (1) results of the content analysis of the last developmental studies mathematics course, the college algebra courses, and the precalculus course at each institution; (2) classroom observation instrument; (3) pre- and post-instruction student and faculty surveys; and (4) college algebra inventory. Contains 12 references. (MKR)

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MINORITIES IN MATHEMATICS: GEORGIA STATE UNIVERSITY ATLANTA METROPOLITAN COLLEGE PARTNERSHIP PROJECT

FINAL REPORT

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Abstract

The Georgia State University - Atlanta Metropolitan College Partnership Project was a collaborative effort to increase articulation between developmental and regular mathematics courses within and between institutions in order to facilitate transfer of students from AMC to GSU. Phases 1 and 2 were funded by the National Center for Academic Achievement and Transfer: A Project of the American Council on Education and the Ford Foundation, Phase 3 by a GTE Focus Grant.

A content analysis was completed for the last developmental mathematics course, the college algebra courses, and the precalculus course at each institution. A classroom observation instrument and pre-/post-instruction student and faculty surveys were developed and used in these same courses.

It was learned that there are some differences in the sequencing of instruction at the two institutions which might pose a problem for students. It was recommended that students transfer at the end of a sequence (e.g., developmental mathematics or college algebra) rather than in the middle of a sequence. The instructional practices employed at both institutions were quite similar. There were differences in levels of classroom interaction and in the breadth of coverage of topics. Both of these differences might affect students transferring from one institution to another.

Other activities sponsored by the project include publication of the College Algebra Inventory, presentation of a dissemination conference, and a faculty curriculum revision retreat.

Introduction & Overview

The Georgia State University - Atlanta Metropolitan College Partnership Project was a collaborative effort between two state-supported institutions in the city of Atlanta. Georgia State University (GSU) is a Ph.D. granting institution; Atlanta Metropolitan College (AMC) is a two-year college granting the associate degree with a predominantly African-American student body (a population underrepresented in mathematics and the sciences). A large number of AMC students transfer to GSU. The primary goal of the GSU-AMC Partnership Project was to study ways to improve the transfer process of students from AMC to GSU, particularly in the area of mathematics. In addition, it was hoped that a secondary benefit of the project would be to improve transfer within institutions from Developmental Studies (non-credit) mathematics courses to regular (credit) mathematics courses.

The project was done in three phases over an 18-month period. Phases 1 and 2 were funded by the National Center for Academic Achievement and Transfer: A Project of the American Council on Education and the Ford Foundation, Phase 3 by a GTE Focus Grant. The activities within each phase were as follows.

Phase 1

- Content analysis of the last developmental mathematics course and the first college algebra course at each institution.
- Development and pilot testing of a classroom observation instrument and student and faculty surveys for use in developmental studies

mathematics, college algebra, and precalculus courses at each institution.

Phase 2

- Development of a Topics in Algebra Inventory.
- Dissemination of the project activities and results to the developmental studies and mathematics faculties at the two institutions via a luncheon with a guest speaker from another of the National Center for Academic Achievement and Transfer projects and hand-outs describing the GSU-AMC project and its results.

Phase 3

- Content analysis of the precalculus mathematics courses at the two institutions.
- Use of the observation instrument and student and faculty surveys to collect data on these three precalculus mathematics courses.

Background & Need

The nation's economy is being threatened by the lack of a literate workforce prepared to meet the needs of the 21st century. By the year 2000, it is estimated that a majority of all new jobs in the United States will require postsecondary education. A very large number of adults will be unable to fill these positions because of their limited literacy skills in reading, writing, oral communication, problem solving, and mathematics (U.S. Departments of Labor & Education, 1988). A recent survey of adult literacy skills (prose, document, and quantitative literacy) found that half of Americans are unable to read and comprehend texts requiring problem-solving skills and to perform quantitative tasks involving two or more sequential operations (Kirsch, Jungeblut, Jenkins, & Kolstad, 1993). Many secondary school graduates are literate in oral and written language, but have very limited literacy in mathematics. "Only half of the nation's students take more than two years of high school-level mathematics" (National Research Council, 1989, p. 38). The problem is especially severe for women and minorities, a smaller percentage of whom attend college. Their low level of participation in mathematics and consequent under-representation in the science, mathematics, and engineering disciplines is a concern to many educators (Syron, 1987) and business professionals (Pollack, 1987). Bridge courses between secondary and college mathematics have been implemented in many institutions as a way to overcome potential college students' deficiencies in mathematics. In most programs, these developmental courses provide remedial instruction in basic algebra and geometry (and in some instances, arithmetic). Unfortunately, however, in most institutions there is limited coordination between the developmental mathematics courses and the entry-level college algebra course, and no coordination between the developmental courses and secondary level mathematics courses (McDonald, 1988) or between the junior colleges that feed the four-year institutions. Thus, the effectiveness of providing a smooth transition for the student is limited.

Georgia State University (GSU), one of the four university-level institutions of the thirty-four member University System of Georgia, is a non-residential institution within an hour's commuting distance of two million citizens in the fifteen-county Atlanta area, more than one-third of the state's population. The enrollment for Fall Quarter 1993 was over 24,000, accounting for approximately one-tenth of all enrollees at accredited colleges in Georgia, both public and private. The average age of undergraduate and graduate students was 26 and 33, respectively. Women constitute the majority of the enrollment (57 percent), African-Americans comprise 17 percent, Asians and Pacific Islanders five percent, and Hispanics two percent. Graduate and professional students account for nearly one-third (30 percent) of the student body. Of the nearly 17,000 undergraduate students in Fall 1992, 11,393 were transfers from 104 other institutions.

Atlanta Metropolitan College (AMC) is a two-year college also in the University System of Georgia. It is located in the city of Atlanta approximately two miles from Georgia State University. The enrollment at AMC is predominantly African-American. Each year a sizeable number of students transfer from AMC to GSU, many of whom have earned Associate degrees at AMC. The University System has an agreed upon core of lower-division courses, facilitating the transfer of credit for students who transfer from one of its institutions to another. Thus, core courses are assumed to be equivalent; students completing an AMC mathematics core course are able immediately to enroll in the next level mathematics course at GSU. However, very few actually declare mathematics as their major at GSU or even take additional mathematics courses. In fact, in the Fall Quarter 1991 of the 354 former AMC students at GSU, only 4 (1%) became mathematics majors.

Statement of Problem

Historically minorities, especially African-Americans, have been underrepresented in the mathematics and science fields. The problem may begin with their experiences with mathematics at the elementary and secondary levels and is certainly exacerbated by their limited participation or success in mathematics courses at the undergraduate level. A large number of minority students do not progress beyond the entry-level course in mathematics (college algebra, for example) and often select majors because they require no more than the core mathematics requirement which all bachelors' degree candidates must complete. In order to attract more minorities as mathematics majors, these students must first experience success in the pre-calculus sequence and enroll in higher-level mathematics courses.

Normally one would expect that the response to this problem of transfer rates and success of transfer students, especially minorities in mathematics, would consist primarily of increased academic support services. These services are being provided by, among others, the Student Support Services Project at AMC which recently received additional funding to work toward increasing transfer rates to four-year schools and by the GSU component of the Atlanta Comprehensive Regional Center for Minorities in Science and Engineering which promotes articulation agreements between two-and four-year schools in the area and provides counseling and tutoring for prospective mathematics and science majors at the two year schools.

However, experience has shown that there is a systemic problem in the sequencing and delivery of mathematics instruction which prevents a smooth transition between mathematics courses offered at transferring and receiving institutions. There is a need for a re-examination and revision of pedagogical processes and content in mathematics courses at both institutions. As a first step in this process, GSU and AMC faculty have been exploring ways to increase the success of transfer students into higher-level mathematics courses by ensuring the compatibility of the GSU and AMC courses beginning in Developmental Studies (pre-college level bridge courses) extending through pre-calculus. A detailed comparison of the content and instructional programs offered at the transferring and receiving institutions would provide a means for studying the transfer process. This would reveal how well sequential courses between institutions support each other and how similar courses address content.

In order to gain this perspective, a three-phase study was conducted to examine the mathematical literacy demands of the collage algebra and precalculus courses at each of these institutions. This mathematical task analysis examined course content and instruction in search of significant factors which influence both: 1) the transfer from developmental to higher mathematics within each institution, and 2) the transfer from mathematics at AMC to higher mathematics at GSU. These data can provide meaningful information about the demands of mathematics courses and the success or difficulty students may have in achieving success in or pursuing mathematics when transferring from AMC to GSU.

Content of Program

The overall goal of the partnership project was to increase articulation between developmental and regular mathematics courses within and between institutions.

Purpose

The purpose of the project was to examine the curriculum and instruction for college algebra and pre-calculus mathematics courses at each institution in search of factors which contribute to the number of minority transfer students from AMC enrolling in higher-level mathematics courses and majoring in mathematics or the sciences at GSU by improving the articulation of the mathematics courses at the two institutions. More specifically, the primary objectives of the research were: (1) content analysis of the course textbooks and syllabi, and (2) survey of students and faculty and observation of classroom instruction for the purpose of analyzing course demands and teaching methods.

Program Design

The project was coordinated by Project Co-Administrators, Dr. Joanne R. Nurss, Georgia State University, and Dr. Barbara Morgan, Atlanta Metropolitan College. It was collaboratively implemented by mathematics faculty members from both Developmental Studies and the Mathematics Department at Georgia State University and Atlanta Metropolitan College (see Appendix A for a list of the faculty members participating in the project). The Center for the Study of Adult Literacy at GSU was the fiscal agent. The project was conducted in three phases, with faculty from both institutions serving on teams to examine the content and instruction of specific courses. Phase 1 was largely a pilot study involving a mathematical task analysis of course content and the development and field-testing of faculty and student surveys and classroom observation instruments for developmental mathematics (GSU/DS 099 and AMC/Math 096) and college algebra (GSU/Math 104 and AMC/Math 101). During Phase 2, the involvement of faculty from Developmental Studies and Mathematics at both institutions was expanded through the dissemination of project activities-to-date and additional participation in the implementation of pilot data. During Phase 3, a mathematical task analysis was conducted on precalculus courses (GSU/Math 126 and AMC/Math 102 & 103) using the revised instruments developed and field-tested in Phase 1.

Methods

The content analysis and observation components were accomplished during Phases 1 and 3. For the content analysis, all sections of the target courses were surveyed to collect course textbooks, syllabi, and final examinations. These were analyzed by a joint GSU-AMC faculty team for topics covered, learning objectives, and expected outcomes. For the observation component, faculty and student surveys were administered and routine classroom observations made for a sample of two sections of each course, one section at each institution. The observations were conducted by trained graduate assistants using the observation instruments developed. In addition, anecdotal field notes were provided about the observations. Variables of class size, class time, and type of instructor (full-time, part-time, or graduate teaching assistant) were considered to get a representative sample. An effort was also made to identify sections of the GSU course that include a subset of students who have transferred from AMC.

Phase 1 began in the summer of 1991. A committee of faculty from both institutions (see Appendix A) met to develop the scope of the project and to develop a time-line for the work. At that time the faculty divided themselves into two teams: the content analysis team and the instrument development team. Each of these teams consisted of a faculty member from Developmental Studies and a faculty member from Mathematics from both institutions. During the academic year 1991-92 the teams met quarterly to review progress and to discuss results to date. The work of each team is reported separately and the results are related in the conclusion.

Content Analysis

The subcommittee on content analysis began a mathematical content analysis of the last developmental courses and the initial college-credit courses in algebra at the two institutions by examining the syllabi, course descriptions, textbooks, and the tests and final examinations. Courses studied were GSU/DS 099 & AMC/Math 096 and GSU/Math 104 & AMC/Math 101. Depth of coverage was analyzed by topic (e.g., real numbers, algebraic expressions, equations and inequalities in one and two variables, problem solving and word problems, functions, exponential and log functions, systems of linear equations and matrices, and polynomials) for each course.

Phase 1

Following lengthy meetings, comparisons between parallel courses at each institution were attempted and it became evident that the last courses in

Developmental Studies were not the same and could not adequately be compared in a narrative form. Similar findings occurred when the initial college-credit courses in algebra were examined.

Following this determination, the subcommittee decided that the most illuminating format for the examination of the courses would be to enumerate the topics covered in the courses (cumulatively), loosely grouped by kind, and then to specify the type of coverage that each course provided each topic. It was also acknowledged that in some cases the situation might arise where a topic was supposed to be covered and was not (or the converse). Thus, coverage was broken down into two aspects: the topic should/could have been covered (investigation of syllabi and text) and the topic was covered (investigation of final examination). It was further noted by the subcommittee that the depth of coverage of a particular topic could range greatly (especially if a spiral approach is utilized). Thus, a means of distinguishing the depth of coverage was developed. Initially, the subcommittee used numerals for depth of coverage as evidenced by test/syllabi and alphabetic designations for coverage as evidenced by final examinations, with three groupings in each case (no coverage, light coverage, heavy coverage by text/syllabi and not needed, implicit usage, explicit usage by examination). It was soon felt that a fourth category indicating assumed knowledge (versus a topic which was not covered because it was beyond the scope of the course) was needed in the text/syllabi investigation. After the initial determination concerning each course, it was further decided to refine the coding to the present form (which implicitly assumes a linear progression in the amount of coverage vis-a-vis "assumed" and which allows for the quantification of the material - a matter not necessarily intended by the subcommittee).

Using the coded topic outline, the subcommittee surveyed the respective departments/divisions concerning each course and reported back the average values, rounded to integers, for each topic for each course based on the division/department responses. It was also reported in greater depth on some statistics for the individual courses (the average, the range of responses, the number of faculty responding with that average). It was felt that this latter could give some indication of the commonality of purpose and content for each course upon which the individual division/department could reflect. It should be noted that there are common final exams (or portion thereof).

Initial findings were reviewed by the faculties in the two departments at each institution and feedback was provided to the team. Based upon this feedback, the final analyses were prepared. These analyses are presented in Appendix B.

Phase 3

Data were collected for the precalculus courses at each institution (GSU/Math 126 and AMC/Math 102 & 103) following the same procedures used for the previous courses. The content analysis team collected course syllabi, textbooks, tests, and the final exam for each of the three courses.

The faculty subcommittee analyzed the precalculus course content following the same approach developed for the analysis of the algebra content. Topics from the course were identified and the depth of coverage for each was determined from the data gathered.

The final content analysis for the precalculus courses was completed and is presented in Appendix C.

Instruction

The instrument development team constructed five instruments to assess course demands: a pre- and post-student survey, a pre- and post-faculty survey and an observation instrument (see Appendix D for the final version of these instruments). A summary of the process follows.

Phase 1

The team met regularly to develop the instruments. Instruments from a similar study in history, political science, English, and biology at GSU (Carson, Chase, & Gibson, 1991) provided some insights into structure and format. The instruments were reviewed by the entire committee and feedback was provided. The intent of the instruments was to provide triangulation from three different perspectives on the demands and requirements of the course: from the perspective of the instructor, from the perspective of the student, and from the perspective of an outside observer.

The observation instrument was organized as a checklist to provide insight into the depth and quantity of mathematical demands placed upon the class, e.g., the amount of homework (25 problems or 5), the content of homework (computational problems or some mix of word problems or other applications) and the scope of homework (is it just from the section completed or does it include previous topics?). The instrument also provided questions on the types and quantity of classroom interactions and classroom organizations. In addition, the observer was encouraged to take notes and to prepare a written narrative following each observation to allow for anecdotal information not accounted for in the checklist.

The student and faculty surveys provided a subset of parallel questions from alternative perspectives. The surveys solicited student and faculty opinions on various aspects of the course including expectations, beliefs about mathematics and the students as mathematics learners, prior knowledge, the difficulty of assignments, and the students' understanding of the prerequisite skills.

A graduate research assistant was trained by faculty on the team to pilot data collection on one section from each course: one developmental mathematics class at each institution (GSU/DS 99 and AMC/Math 96) and one college algebra mathematics class (GSU/Math 104 and AMC/Math 101). At the beginning of the quarter the pre-faculty and pre-student surveys were administered. During the quarter eight observations of each class, consisting of four sets of two concurrent observations each, were conducted. Random observations by faculty were used to assure reliability. At the end of the quarter the post-faculty and post-student surveys were administered.

Results from the five instruments were analyzed by a team of two graduate assistants, including the graduate assistant who collected the data, and faculty members of the instrument development team. For the observations and pre- and post-faculty surveys, results were analyzed by individual faculty member, by program

(DS Mathematics faculty members or Mathematics Department faculty members), and by institution (GSU or AMC). For students, class data were analyzed for all students completing the pre- and post-survey and for students who completed both pre- and post-surveys; i.e., students who remained in the course the entire quarter. These results were further analyzed by individual, by institution, and by program and are reported below.

Phase 3

Phase 3 began with the pilot survey and observation instruments being revised to eliminate ambiguous items, to insure parallel items, and to develop uniform item responses for analysis. Two graduate students were trained by members of the instrument development team to conduct observations. Data were collected on one section of the final developmental studies mathematics course (GSU/DS 099), one of college algebra (GSU/Math 104), and two sections at each institution of precalculus (GSU/Math 126 and AMC/Math 102 & 103).

Data were compiled and initial analysis, similar to that performed for the Phase 1 pilot study, was performed. The data gathered from the observation instrument and pre- and post-faculty and student surveys provided insights of the trends from more than one perspective. The data were also analyzed by institution, by program (GSU/Math 126 and AMC/Math 102 & 103), and by instructor. The graduate students summarized trends or themes which were noted from their observations.

More detailed analyses of the data were performed. Examples of the findings are described below.

Results

The results of the two aspects of Phases 1 and 3 are presented in the following section.

Content Analysis

Phases 1 & 3

At Georgia State University there is a sequence of three Developmental Studies mathematics courses, but Atlanta Metropolitan College had only two courses in their Developmental Mathematics sequence when the study was completed. At GSU, there is one college algebra course and one precalculus course preceding the calculus courses, while at AMC there is one college algebra course, one transitional college algebra/precalculus course, and one precalculus course preceding the calculus courses. These course sequences can be summarized as follows:

AMC: DM1, DM2, CA, PC1, PC2, Calculus
GSU: DM1, DM2, DM3, CA, PC, Calculus

In the first phase of the project, the content analysis team considered the last developmental mathematics course and the first college algebra course at each institution. In the third phase of the project, the team considered the other courses that precede the calculus courses. The content analysis team also tried to evaluate the depth of coverage for each topic as reflected by the textbooks and examinations.

Based on these observations, the following conclusions were drawn:

1. While the two-quarter AMC developmental mathematics sequence covers material prerequisite to a college algebra course, the three-quarter GSU developmental mathematics sequence covers this same prerequisite material and also previews topics contained in a college algebra course.

2. It is difficult for students to transfer from one institution to another in the middle of a sequence of courses. They either have to repeat material, or skip material. These difficulties can be avoided by transferring at the beginning or end of a sequence, that is after developmental mathematics or college algebra, or just before the calculus courses. For students who do not transfer at such times, detailed course descriptions by topic should be supplied as an aid to advisement. When cross registration is available, it should also be considered by these students in order to complete a sequence already begun at one institution.

3. Fewer topics are covered in the three mathematics courses preceding the calculus courses at AMC than in the corresponding courses at GSU.

4. Although textbooks used at AMC have a lower level of difficulty from those used at GSU, these same differences are not apparent on course examinations.

5. Students who complete their mathematics requirement at AMC before transferring to GSU may have less breadth of coverage in mathematics. For example, a student in a degree program that requires ten hours of mathematics could take five hours of college algebra and five hours of a transitional college algebra/precalculus at AMC, but would only take five hours of college algebra at GSU. The other five

hours of mathematics at GSU would typically be chosen from statistics, precalculus, or liberal arts mathematics.

Instruction

Phase 1

Due to the very small sample, it was not the intent of this phase to conduct an in-depth analysis of the data. Rather the goal of the committee was to develop and pilot instruments. However, a sample item analysis provides some interesting information as to the potential richness of the research in Phase 3.

To illustrate this, the amount of time expected to be spent on homework and the actual number of homework problems assigned will be discussed below. Table 1 shows the expectations of students about the amount of time they will need to spend on homework for the course before the course begins.

Table 1

Amount of Time Students Expect to Spend on Homework Before the Course (in percents)

	<1/2 hr	1/2-1hrs	1 - 2hrs	>2 hrs
All Students	9	46.5	39	5.5
Students at 4-year institution	9	49	36	6
Students at 2-year institution	9	44	42	5
Students in credit institution	11	44	36	9
Students in developmental math courses	7	50	43	0

Notice that most students in credit mathematics courses tended to have lower time expectations than students in the developmental courses. And, more students at the two-year institution tended to think they would need slightly more time on homework than those at the four-year institution. Table 2 provides the results from students after they had completed the course.

Table 2

Amount of Time Students Reported Spending on Homework After the Course (in percents)

	<1/2 hr	1/2-1hrs	1 - 2hrs	>2 hrs
All Students	13.5	44	37	6
Students at 4-year institution	16	47	28	9
Students at 2-year institution	11	41	46	3
Students in credit institution	14	50	31	5
Students in developmental math courses	13	37	42	8

Notice that more students in all categories reported spending less than half an hour on homework than what was expected before the course began. Also, for students in the credit-mathematics courses, there was a tendency to spend less time on homework than they expected and students in the developmental courses tended to spend more time on homework. No clear pattern was seen between the two- and four-year institutions.

It is impossible to make definitive statements based on such a small number of classes. However, it is interesting to note that in this particular case the discrepancy between expectations and actual experience is larger between the programs than between the institutions.

Finally, Table 3 provides average numbers of homework problems given at the two institutions.

Table 3

Average Number of Homework Problems Assigned Based on Classroom Observations

All Faculty	29.8
Faculty at 4-year institution	22.5
Faculty at 2-year institution	37.1
Math Faculty	27.4
Developmental Faculty	32.3

There is some consistency between the time spent on homework and the number of problems assigned. Faculty in the mathematics departments tended to give slightly fewer homework problems than faculty in the developmental departments. Again, no definitive statement can be made, but on a larger scale this type of information may provide some information on trends and procedures that help or interfere with transfer within and across institutions.

Phase 3

The results being reported identify trends which were obtained from the analysis of data gathered from the revised faculty and student surveys and the revised classroom observation instrument. As indicated above, data were analyzed by institution, by program, and by instructor. This data-slicing, along with the triangulated perspectives offered by the student and faculty survey responses and observer observations, helped to provide a more complete picture of the instructional program at each institution. The most complete analysis of the data occurred by institution and these results are reported in the following pages. Thus, data from each of the three instruments (faculty survey, student survey, and observation instrument) were collapsed into an institutional response and then compared. All course data collected during this phase were analyzed. In addition to the two sections of precalculus courses at each institution (GSU/Math 126 and AMC/Math 102 & 103), one section of the last developmental mathematics course and the college algebra course taught at GSU (GSU/DS 99 and Math 104) were included in the analysis. It is recognized that a cleaner comparison would have resulted by only including data from comparable courses at each institution, yet the decision was made to analyze all available course data. Therefore, it should be recognized that the results being reported reflect a larger institutional sample of courses from GSU than from AMC.

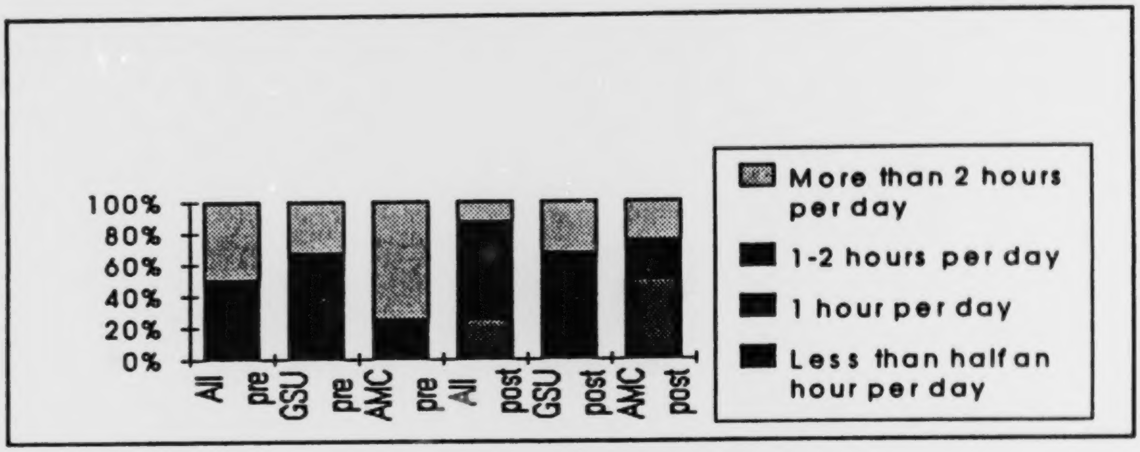
Since a sample size of only one section (instructor) for each course at each institution is too small to draw broad generalizations, the trends being reported provide examples of the kind of information which can be obtained from such a study.

Thus the reported findings can not be considered conclusive, but point to interesting themes about instructional practice. The trends have been organized and are reported within the following categories: homework, tests, classroom interaction, student preparation, and student expectations.

Homework. The amount of homework assigned on average varied at each institution. The average number of problems assigned at AMC was nine while the average number at GSU was 32 problems. Faculty generally expected homework to take longer than did students. Initially 50% of the faculty reported the time required for homework to be between one and two hours while 50% initially reported a time of more than two hours. Although for post-course responses, 25% reported between 1/2 and 1 hour, the majority of the faculty (76%) reported that homework would require more than 1 hour. The faculty from AMC and GSU revised their expectations after the course, shifting to less time required for homework (see Table 4).

Table 4

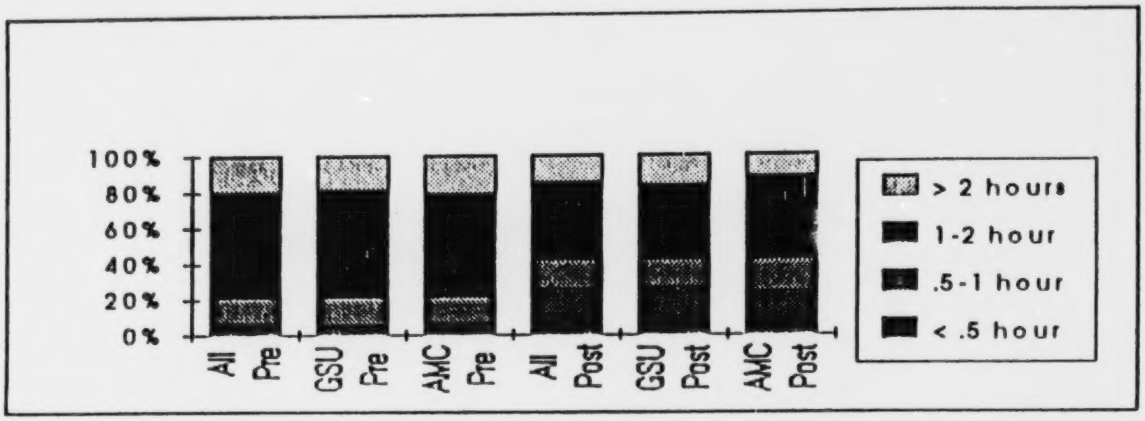
Time Required for Homework: Instructor's View



Students' initial expectations regarding time required for homework were somewhat different. Although a majority of the students (79%) initially felt the homework would take more than 1 hour, a considerable number expected homework to require less than 1/2 hour (6%) or between 1-2 hours (16%). The post-course responses revealed that homework required less time than expected, with 26% taking less than 1/2 hour for homework, 15% taking between 1 and 2 hours, and 59% taking more than 1 hour for homework. A comparison of Tables 4 and 5 illuminates the differences between faculty and student responses to the question of time required for homework.

Table 5

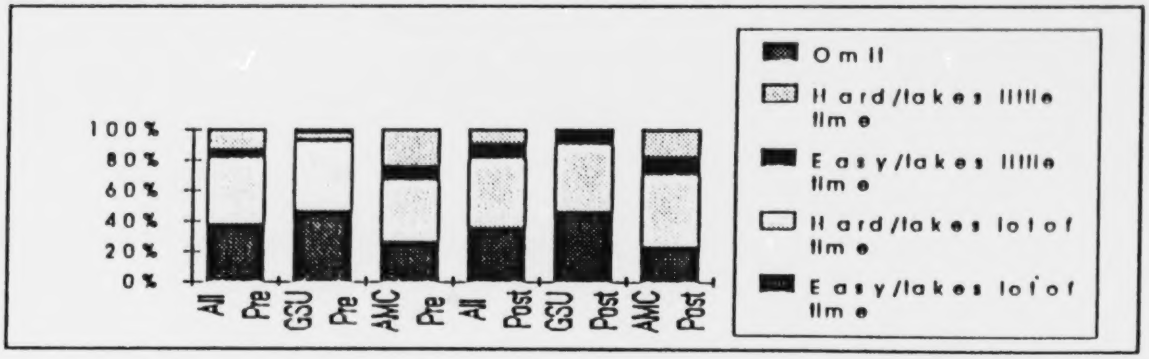
Homework Time



Generally, students maintained their expectation that homework would be time consuming although the level of difficulty of the assignments was considered both easy and hard as reflected in Table 6.

Table 6

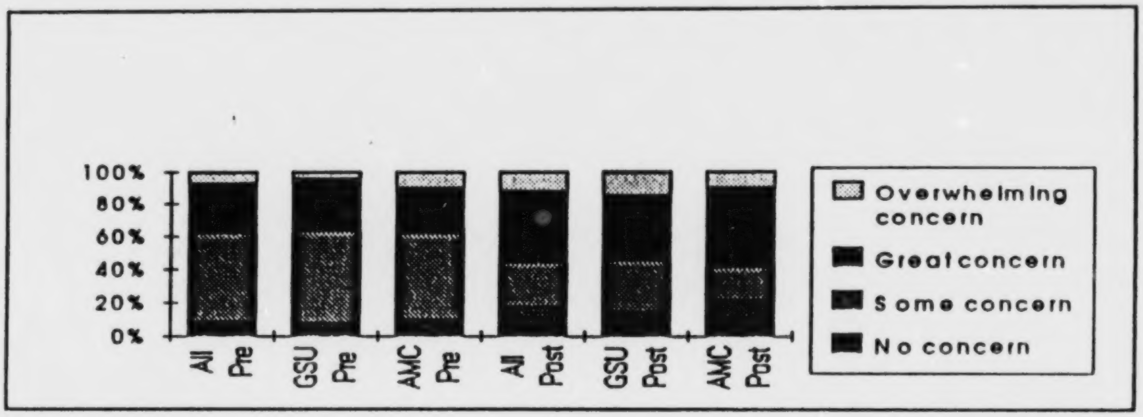
Homework Difficulty



Students maintained a concern over the time required to complete homework throughout the course (see Table 7).

Table 7

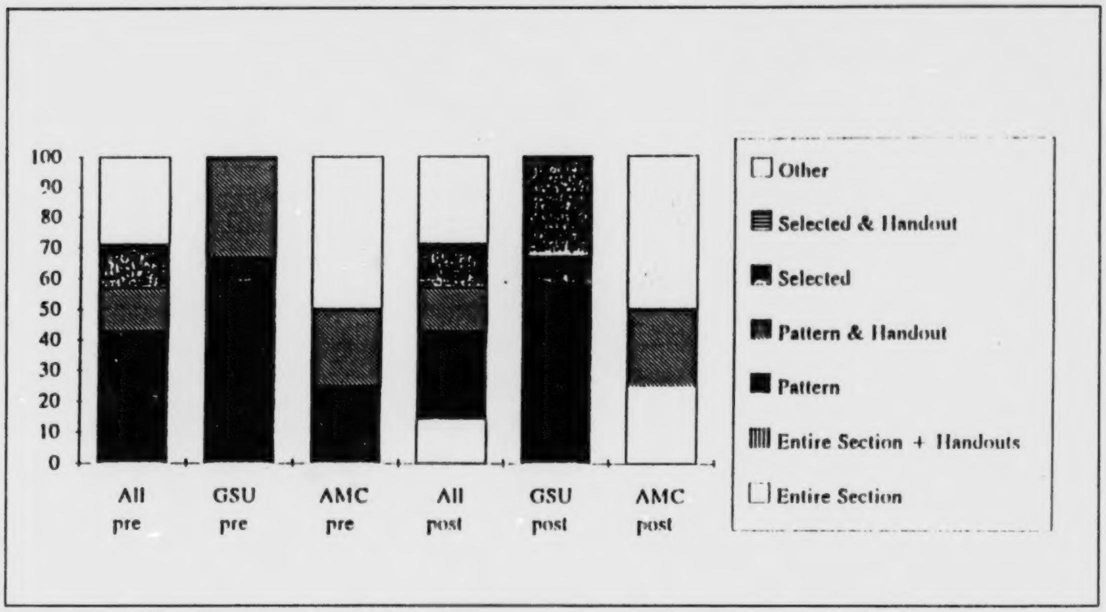
Time Required for Homework: Student's View



There were many similarities about the manner homework was covered and assigned at both institutions. The majority of the faculty discussed it with students, allowed students to select the problems to be reviewed, explained the problems rather than just providing the answers, and assigned homework from the text (see Table 8).

Table 8

Format of Homework Assignments



Tests. The expectations for the test structure were quite similar for faculty and students. The vast majority of faculty at each institution reported anticipating and giving primarily "show your work" items on the tests (91%-89% at AMC; 90%-94% at GSU; 91%-91% for both AMC & GSU). Students reported a high expectation and occurrence that many or all of the test problems would be of the "show your work" variety (85%-98% at AMC; 89%-89% at GSU; 88%-93% for both AMC & GSU). Tables 9 and 10 illustrate these similarities.

Table 9

Expected Test Questions: Student's View

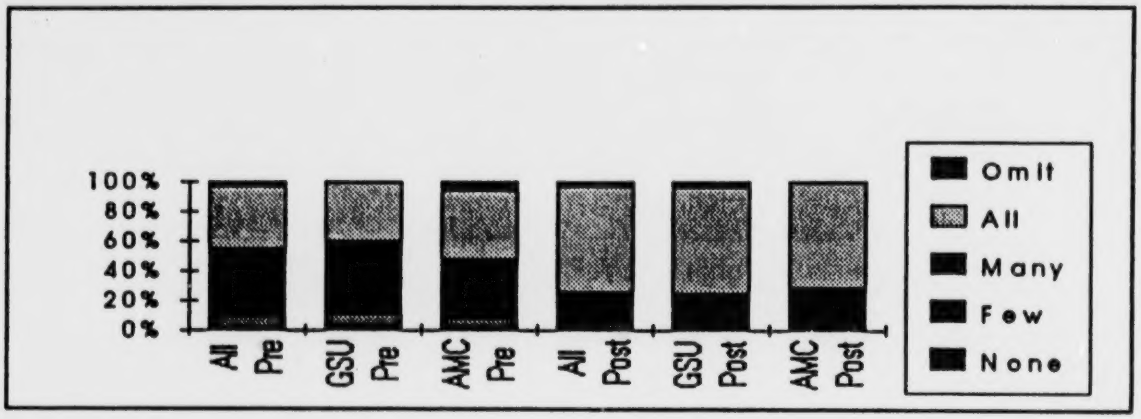
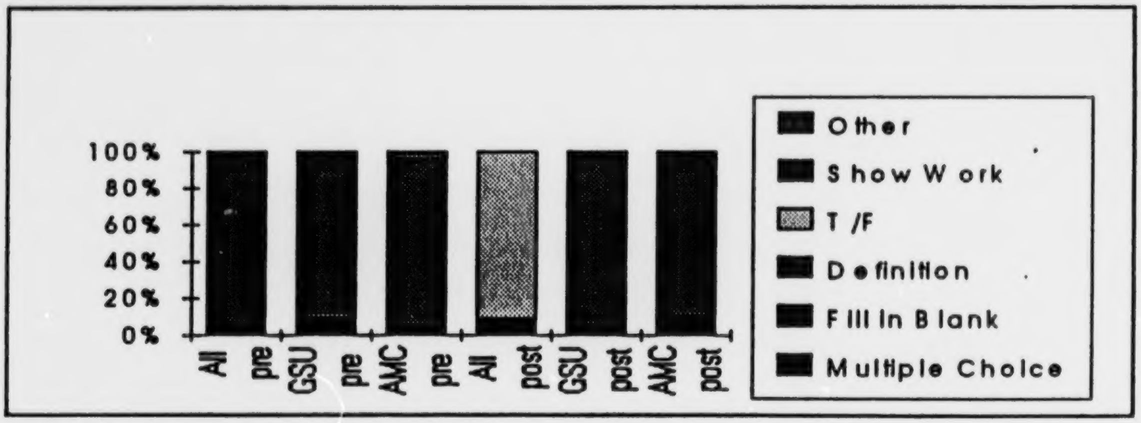


Table 10

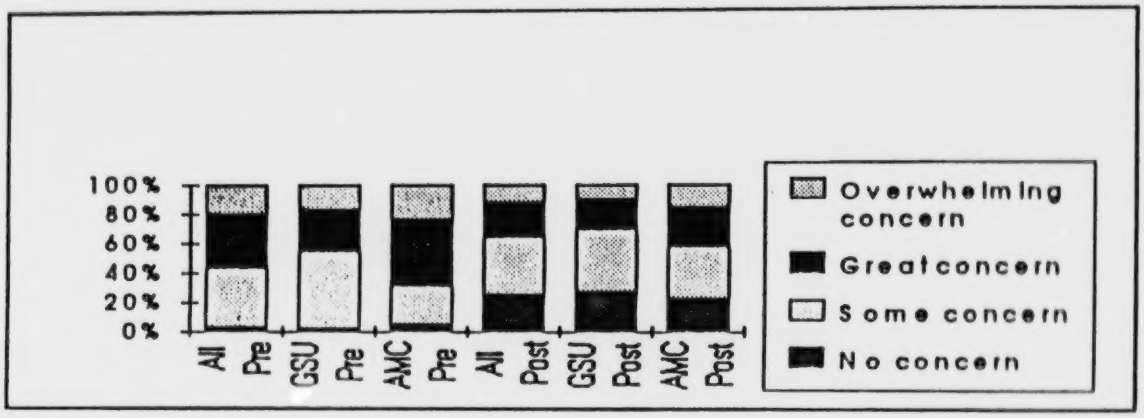
Types of Test Questions



As is shown in Table 11, the time necessary to complete exams and quizzes surfaced as being a great or an overwhelming concern for students at both institutions, dropping only slightly from students' initial expectations to those expressed at the completion of the course (55%-49% at AMC; 53%-43% at GSU).

Table 11

Time to Finish Quizzes/Exams



Classroom Interaction. Most students at each institution anticipated that their instructor would show them how to solve problem outside of class, yet the percentage reporting this actually occurring was much lower (86%-44% at AMC; 67%-28% at GSU). A similar trend was reported for working with other students in class (90%-54% at AMC; 60%-37% at GSU). Tables 12 and 13 display these comparisons.

Table 12

Expectation of Instructor Outside of Class

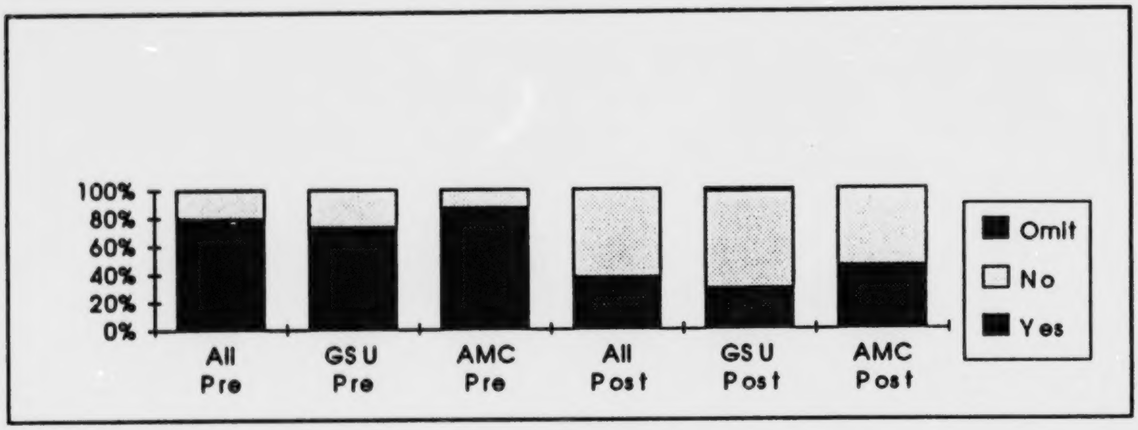
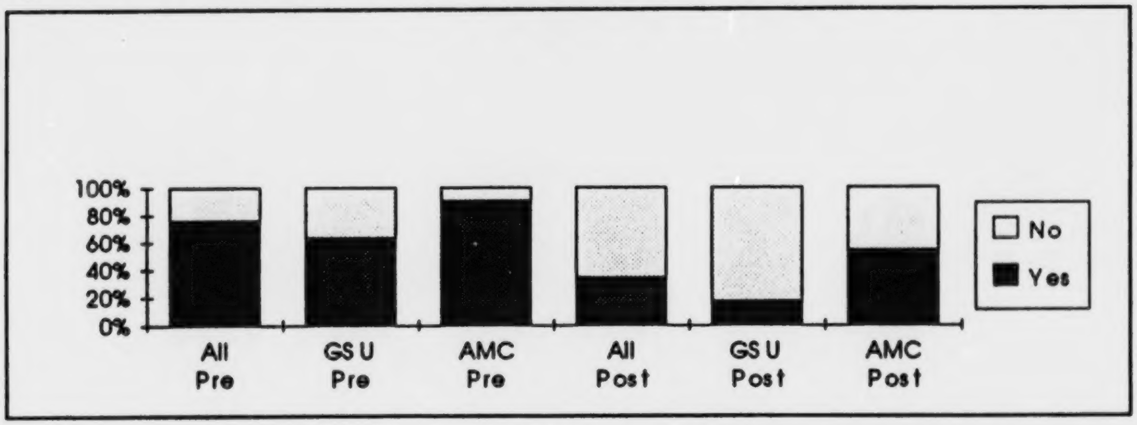


Table 13

Expectations of Working with Other Students in Class



Faculty at each institution varied the instructional methods to some extent over the course of the study. Specifically, they indicated using different methods of presentation than anticipated. At GSU 25% reported using lecture, instructor examples and student solutions instead of instructor problems, questions and discussions. At AMC 25% reported using lecture, related examples, and student solutions instead of the anticipated lecture and instructor examples and student solutions. Although all faculty anticipated following the text very closely, faculty at AMC reversed its use. Seventy five percent planned to follow the text closely, however only 25% actually did. At GSU 100% of the faculty anticipated following the text, yet 25% deviated from this practice. This information shown in Tables 14 and 15.

Table 14

Typical Classroom Presentation

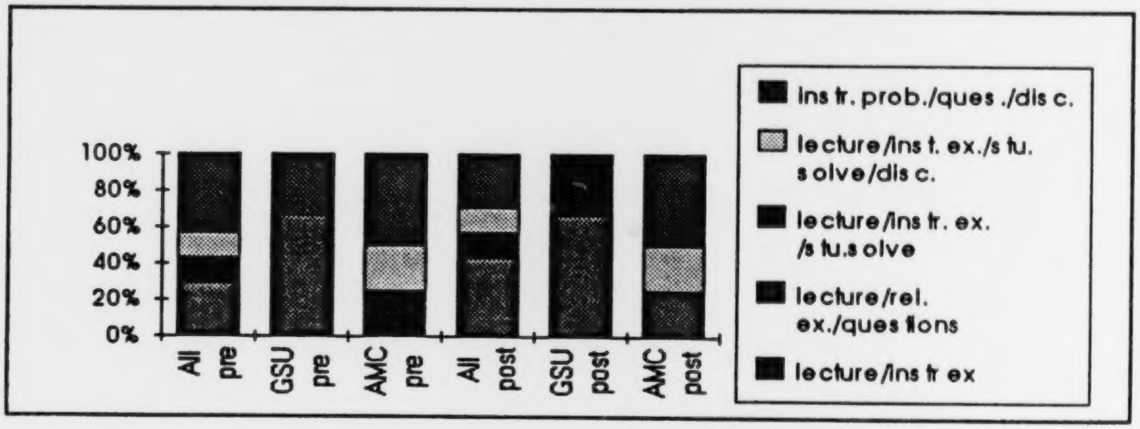
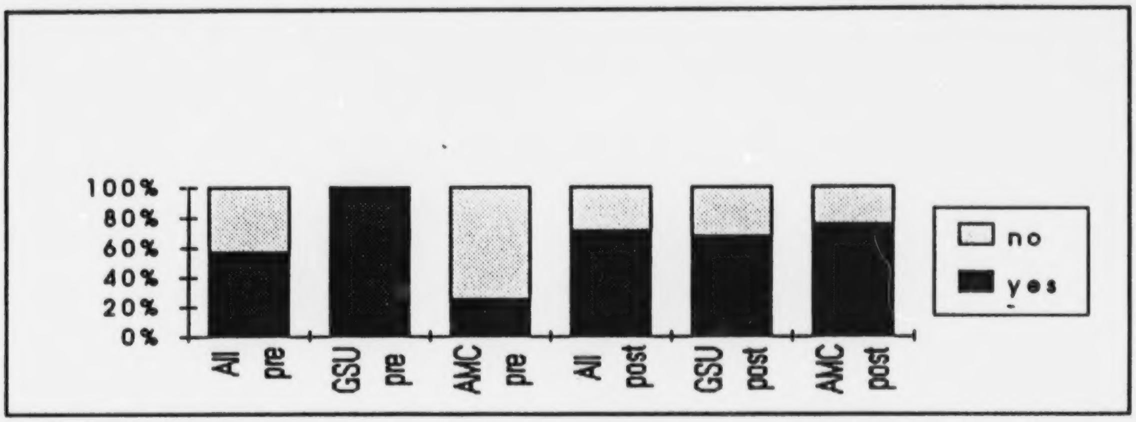


Table 15

Presentations Relationship to Text



Observations revealed similar instructional practices for all faculty. The majority of the faculty probed for difficulties with homework or previous material and asked students to predict outcomes, synthesize ideas, and recall previous information. Faculty also requested or demonstrated a variety of strategies to solve problems. This was also reported as being a common practice by students and faculty. Tables 16, 17, and 18 illustrate this information.

Table 16

Probes for Specific Causes of Difficulties with Homework

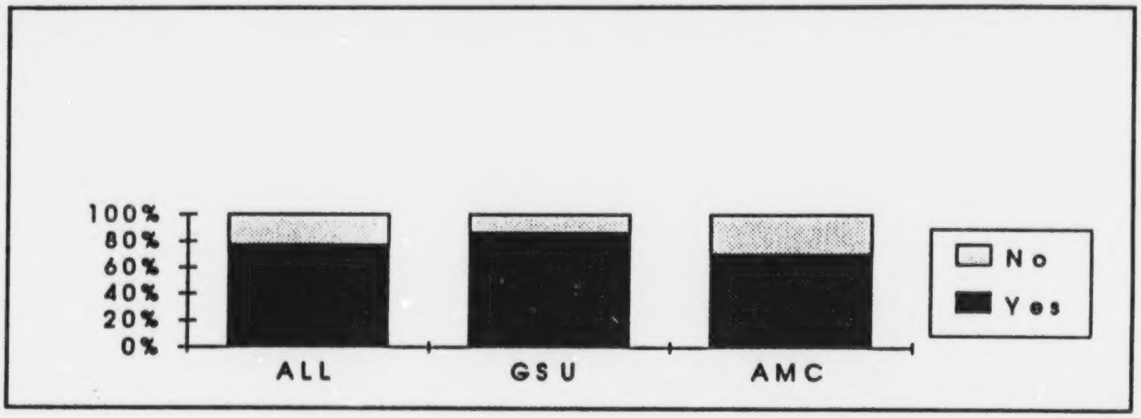


Table 17

Instructor Asks Students to Predict Results

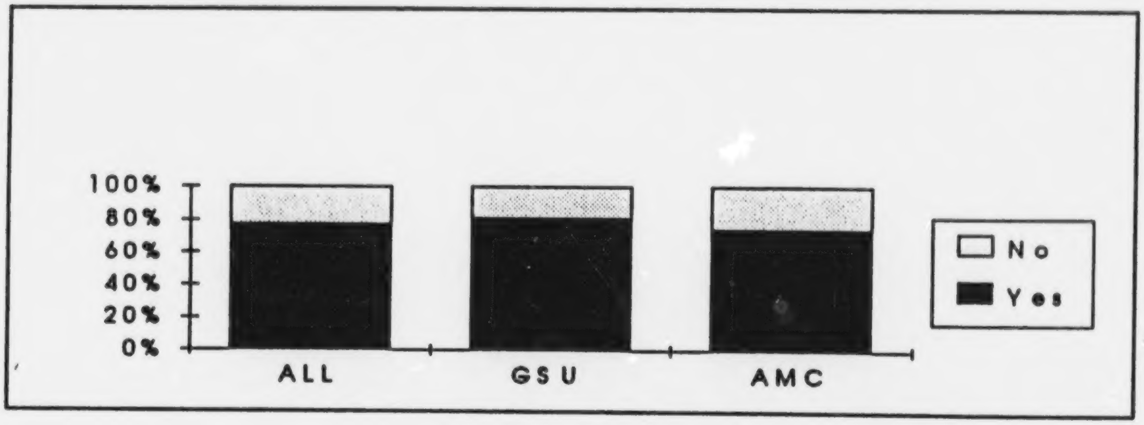
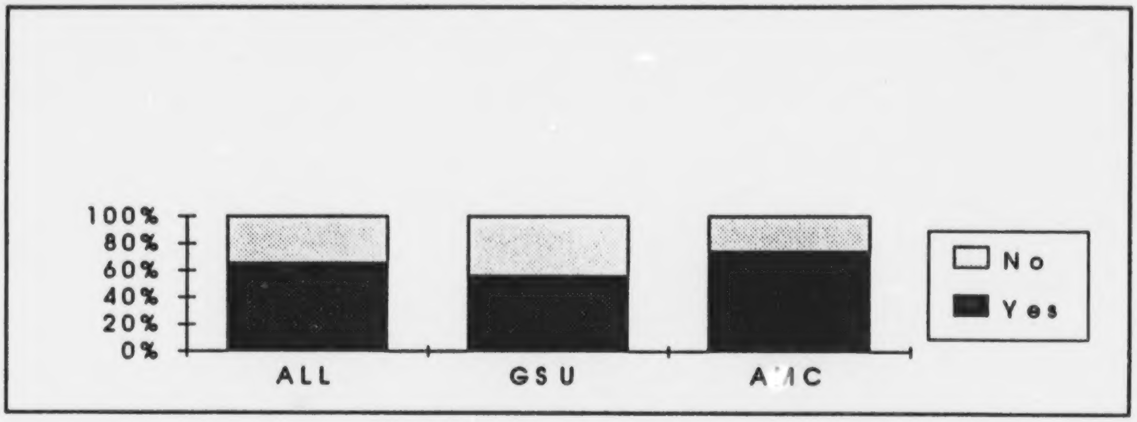


Table 18

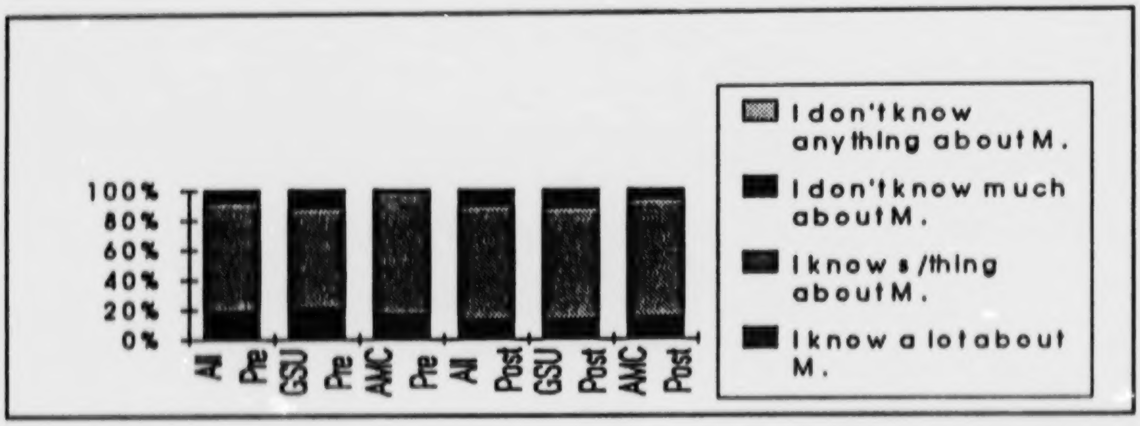
Instructor Requests a Variety of Strategies to Solve a Problem



There were differences observed in the overall classroom atmosphere at each institution. Although student participation was encouraged at both institutions, students at GSU were less actively involved in asking questions and in class discussion, while the majority of students at AMC were involved in this manner. Students at AMC were less involved in note-taking than students at GSU (see Table 19).

Table 19

Use of Non-text Materials to Solve Problems



Student Preparation. Faculty and students had differing views regarding the level of the students' mathematics preparation. The majority of the students reported having some or a lot of knowledge about mathematics (98%-93% at AMC; 90%-91% at GSU). Most reported taking their last math course from 1 to 3 years ago (97% at AMC; 92% at GSU) and that their last math course was in the 12th grade or in college (100% at AMC; 85% at GSU). Yet a slightly smaller percentage of faculty felt that 50% -100% of the students were adequately prepared for the course (100%-75% at AMC; 75%-75% at GSU). For those students who were considered inadequately prepared, faculty considered lack of necessary skills in previous mathematics courses (100%-100% at AMC; 75%-75% at GSU) and lack of prerequisites (100%-75% at AMC; 50%-75% at GSU) somewhat or a major factor contributing to poor performance (see Tables 20 & 21).

Table 20

Student Self-descriptions

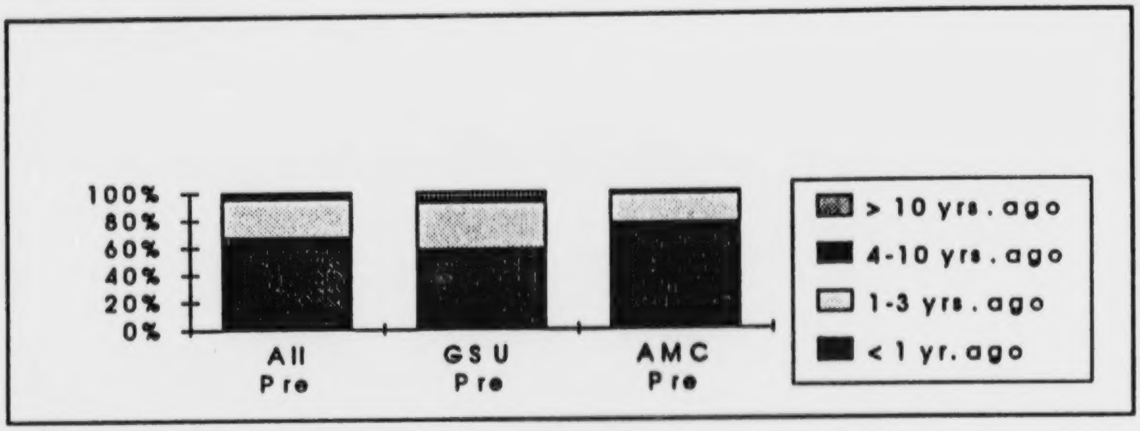
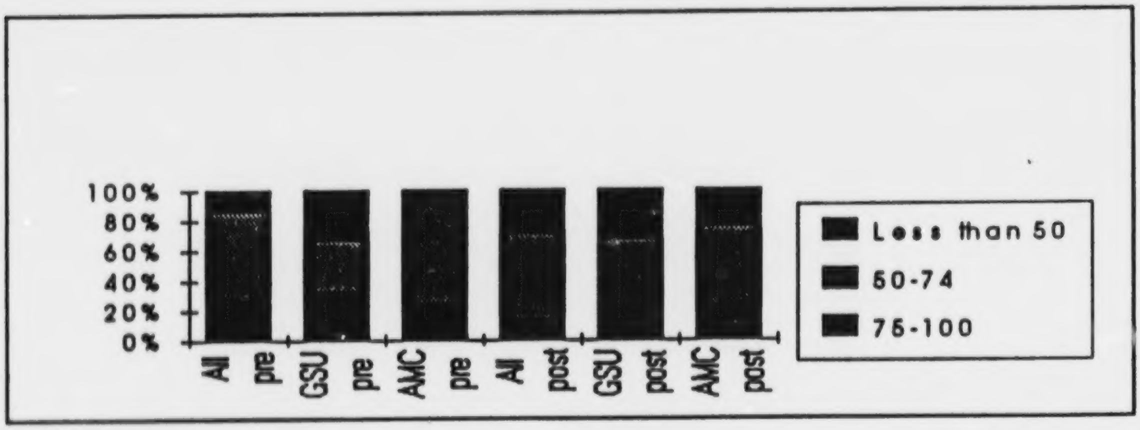


Table 21

Last Mathematics Course Taken



The majority of the faculty viewed students' grasp of prerequisite concepts to be average or excellent (100%-75% at AMC; 75%-100% at GSU). Similar views were expressed for students' grasp of course requirements (100%-100% at AMC;

100%-100% at GSU). In contrast, all felt the students' grasp of prerequisite computational skills was average or poor (100%-100% at AMC; 100%-100% at GSU) and most considered students' problem-solving abilities to be the same (100%-75% at AMC; 100%-75% at GSU). Tables 22 through 26 depict these differences in perceptions from faculty and student points of view.

Table 22

Relationship of Lack of Necessary Skills to Poor Performance

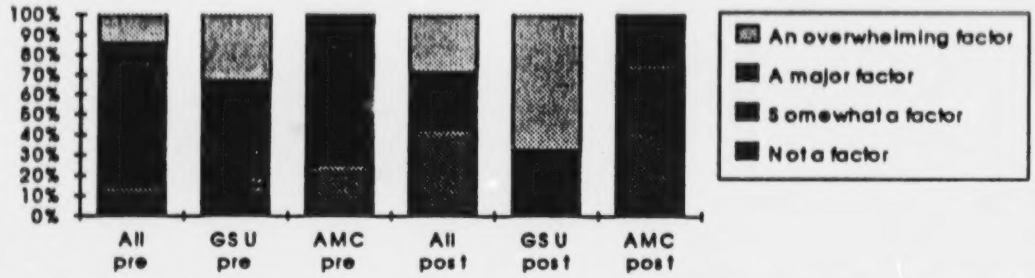


Table 23

Relationship of Lack of Prerequisites to Poor Performance

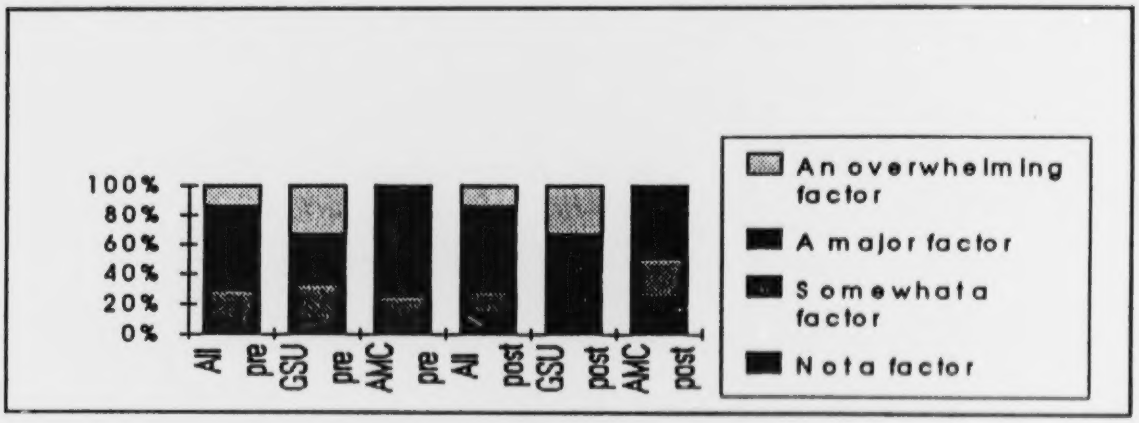


Table 24

Student Grasp of Prerequisite Concepts

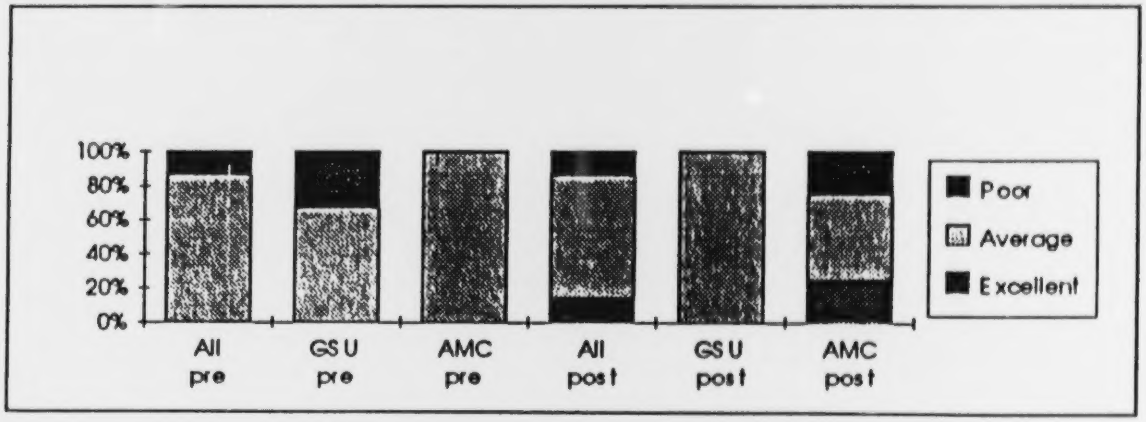


Table 25

Student Grasp of Prerequisite Computational Skills

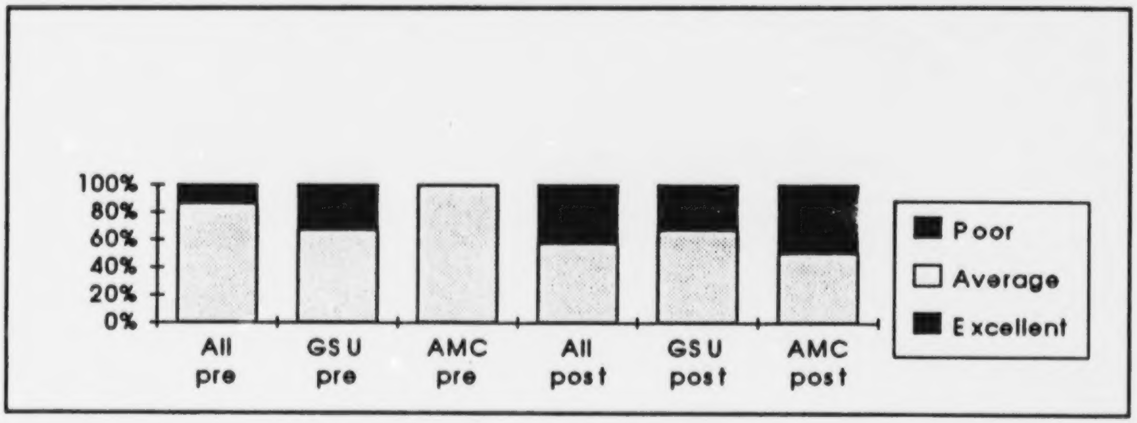
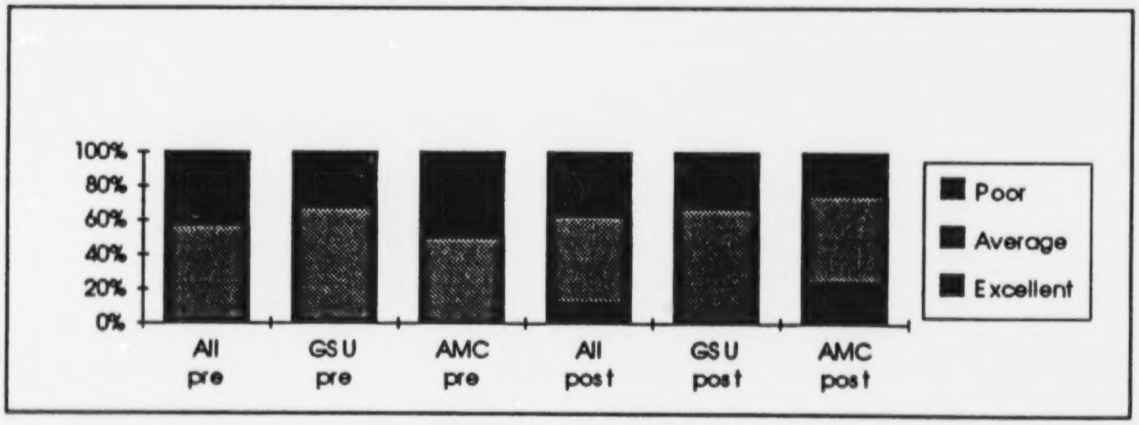


Table 26

Student Grasp of Problem Solving Skills



Student Expectations. Most students expressed the belief that in mathematics there is usually one right answer (81%-80% at AMC; 81%-73% at GSU) although this opinion decreased slightly at GSU. Most also expressed the belief that most mathematics problems can be solved in less than 10 minutes (69&-80% at AMC; 73%-81% at GSU). However, the vast majority responded that there is more than one way to solve a mathematics problem (90%-98 at AMC; 88%-90% at GSU; see Tables 27, 28, and 29).

Table 27

Belief That There is Only One Right Answer in Mathematics

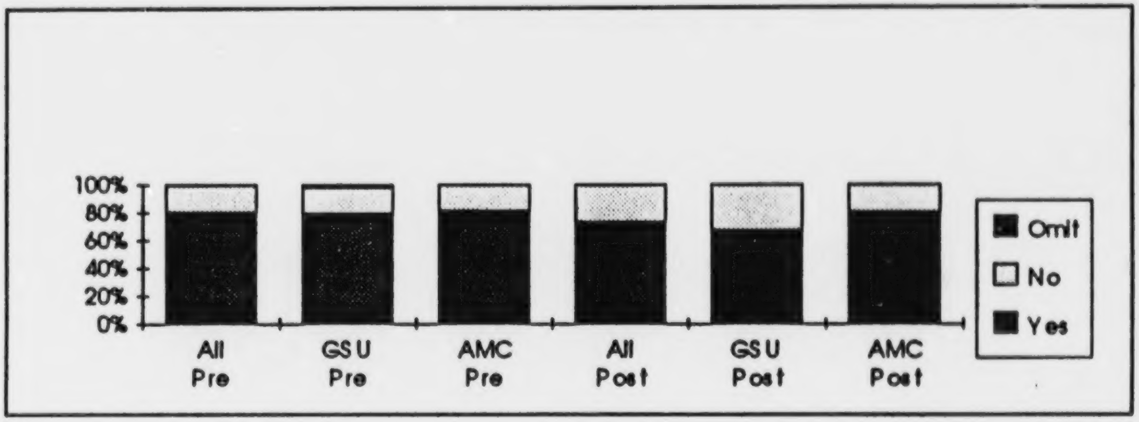


Table 28

Belief That Most Mathematics Problems Can Be Solved in Less Than 10 Minutes

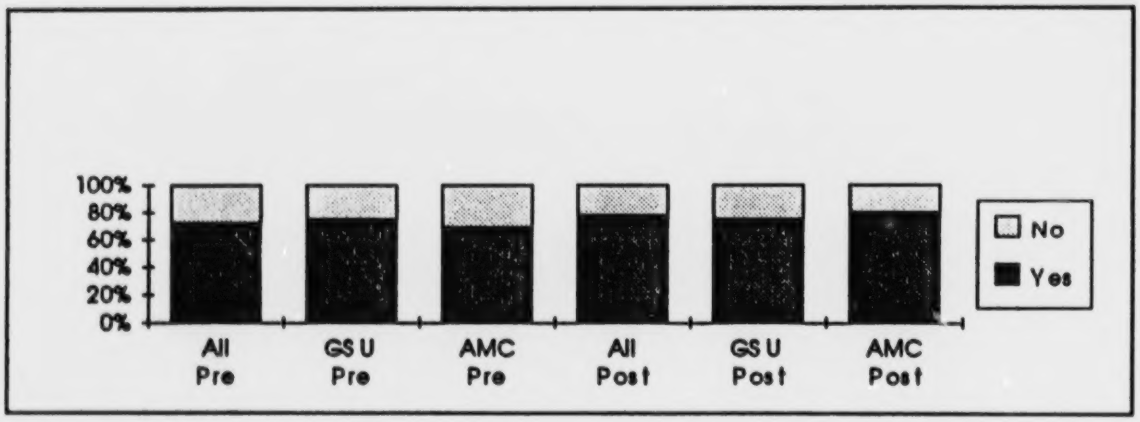
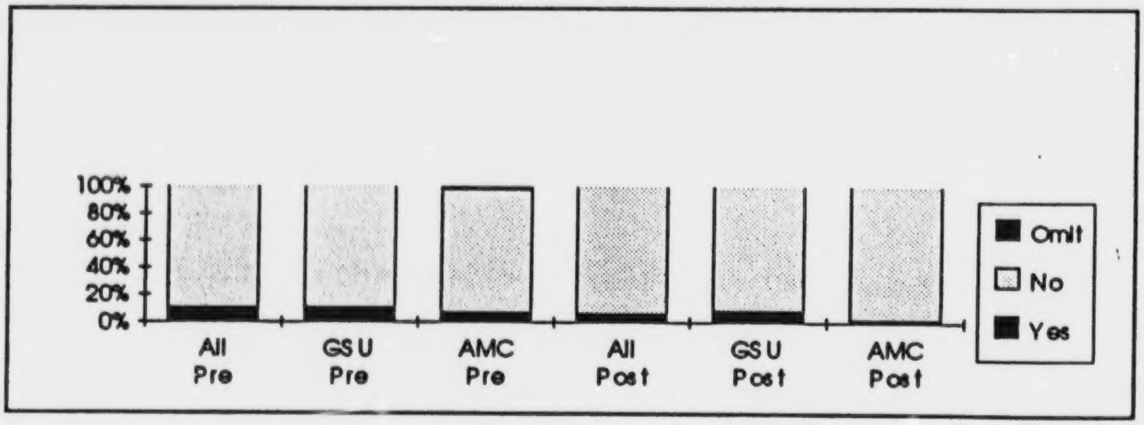


Table 29

Belief that in Mathematics There is Usually Only One Way to Solve the Problem



Nearly one-half of the students considered their course load (43%-52% at AMC; 45%-41% at GSU) and responsibilities outside of class, i.e. work or home (59%-67% at AMC; 58%-61% at GSU), of great or overwhelming concern (see Tables 30 and 31). Faculty, in contrast viewed course load and non-academic factors as only somewhat a factor in poor performance (see Table 32).

Table 30

Concern About Heavy Course Load

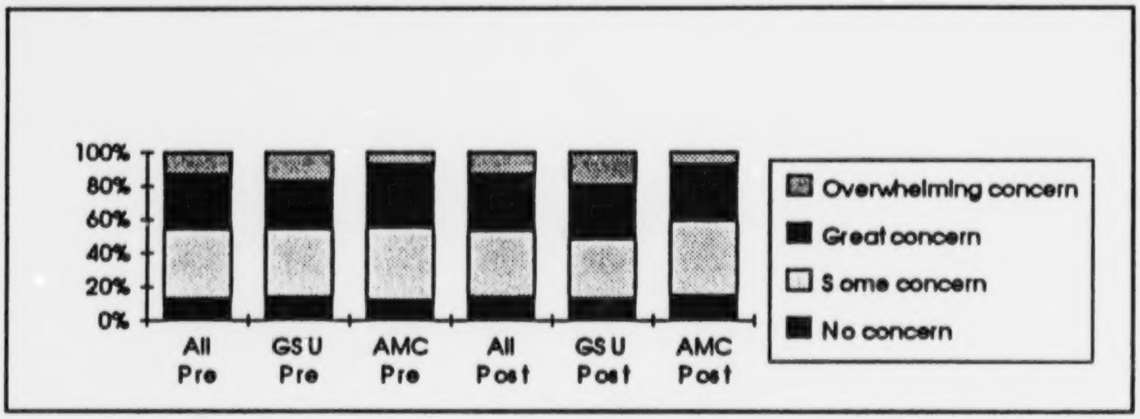


Table 31

Concern About Outside of Class Responsibilities

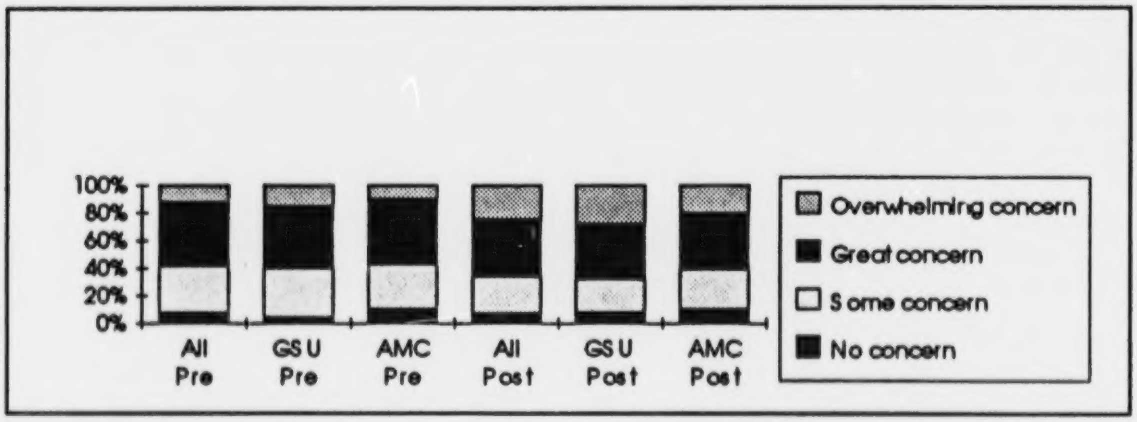
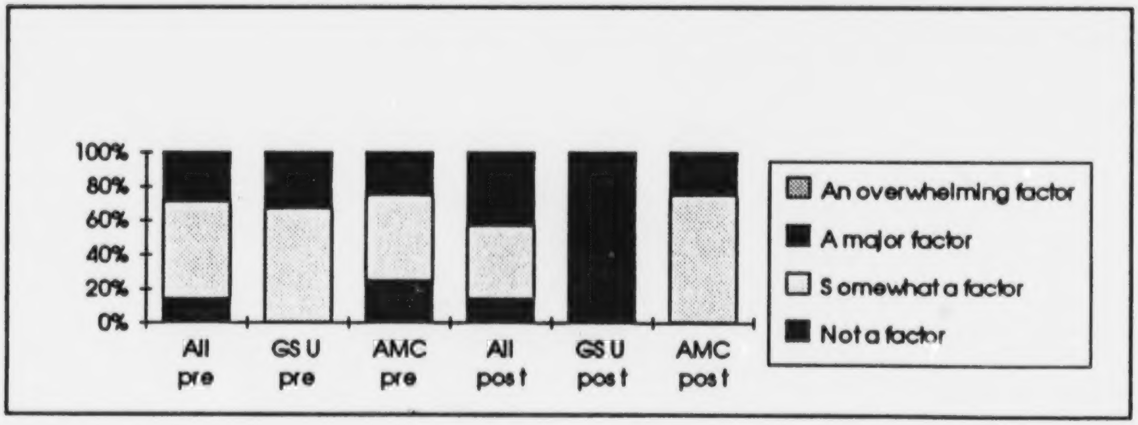


Table 32

Relationship of Heavy Course Load to Poor Performance



Faculty perceptions of student expectations regarding grades, required work, and content coverage were mixed and responses changed from beginning to end of the course. Some responses were evenly divided. Fifty percent of the AMC faculty perceived students as underestimating the grades, work, and coverage and 50% perceived an underestimation of just grades. Initially 50% of the AMC faculty perceived none of these categories as being overestimated by students. This, however, grew to 100% as a final response. Half of the AMC faculty initially perceived student expectations for all categories to be reasonable. This fell to one-quarter by the end of the course.

Initially, some GSU faculty members perceived students as underestimating a combination of grades, work required and content coverage, yet by the end of the course half of the faculty felt only work was underestimated by students. Half of the faculty maintained a perception of grades being overestimated, but an equal percentage shifted (25%) from initial content coverage to work at the end of the course. Although the percentage responding decreased, GSU faculty generally saw students as not having reasonable expectations for any of the categories (see Tables 33, 34, and 35).

Table 33

Student Expectations Seem Underestimated

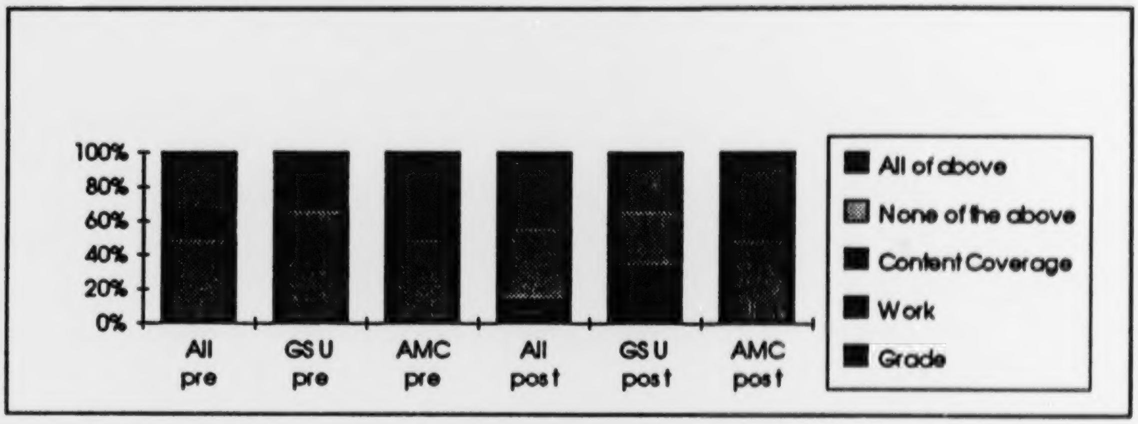


Table 34

Student Expectations Seem Overestimated

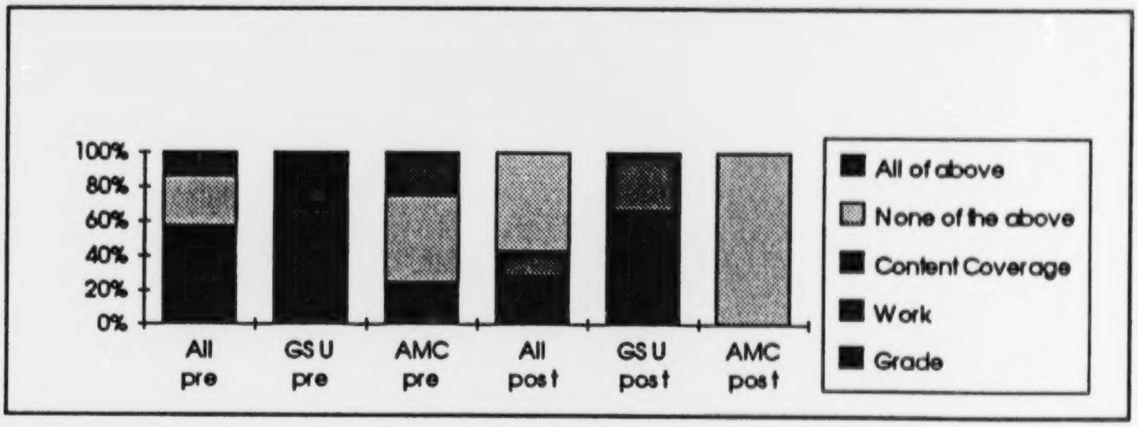


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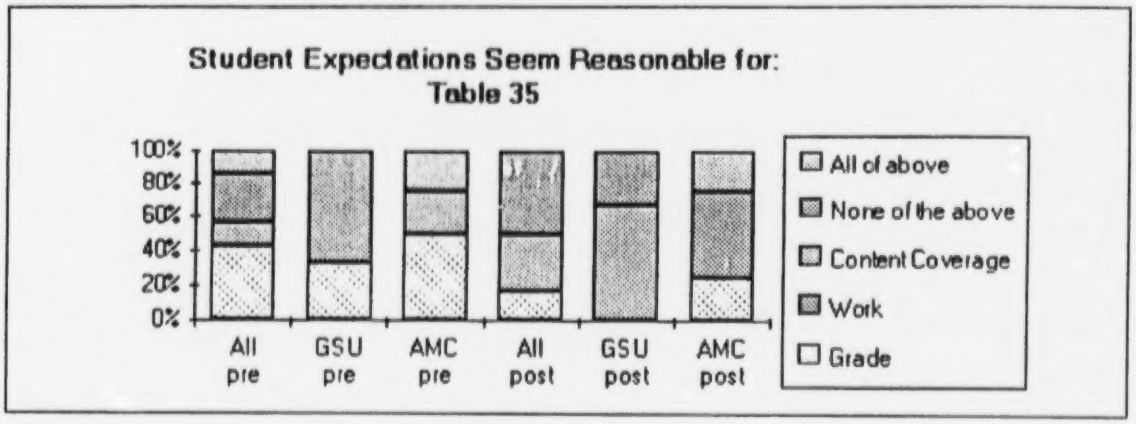
Table 35

Student Expectations Seem Reasonable



Discussion

The varied responses from the pre- and post-faculty surveys indicate that the process of faculty engaging in a formal reflection of their instructional practice and their beliefs about specific aspects of their courses is not a common experience. The instructors participating in this study are experienced teachers and hence one may assume that their experience in the profession would make them more informed and certain about their instructional practice and typical student profiles. Yet the changes in their pre- and post-responses to certain items implies otherwise. Some of the variations, such as changing the anticipated method of presentation, can be assumed to result from the instructional decisions made in responding to student needs. Yet it is curious to note that some of the pre- and post-responses to items are completely reversed. This is seen in responses such as those referring to the percentage of students who are prepared and the percentage who leave previous courses without the necessary skills as shown in Table 22.

There also appeared to be an inconsistency between student expectations and instructor expectations for particular aspects of instruction. An example of this is seen in the students' high expectation of working with other students in class. However this did not typically occur.

Faculty and student perceptions and beliefs about students' mathematical knowledge and reasons for poor performance were quite different. Students typically viewed themselves as mathematically knowledgeable, yet viewed non-academic responsibilities as playing a significant role in contributing to a lack of success. In contrast, faculty viewed students as being less knowledgeable about mathematics and viewed a lack of specific mathematics skills as contributing significantly to students' lack of success. Unlike the students, faculty did not view non-academic responsibilities as major impediments to student success. Thus, faculty perceptions of students as mathematical learners and the factors contributing to lack of success were quite different than the students' perceptions of themselves.

The fact that more students believed at the end of the courses that most mathematics problems can be solved in less than 10 minutes would seem to indicate that the kinds of problems students are typically given reinforces the belief that problem solving is not time consuming. Yet, this is a false impression, particularly when mathematical problem solving is taken beyond the textbook and encountered in a real world context. Thus, students would benefit from experiencing non-routine problem-solving activities which require more thought and time to solve.

It is particularly interesting to note that most students expressed significant concern over their course load and their outside-of-class responsibilities, while faculty viewed these factors as only somewhat significant. This concern among students may be more common at urban and commuter institutions which have larger working and older student populations such as AMC and GSU. However, these concerns should be addressed in advising incoming and transferring students and should play a role in making policy decisions to provide adequate support for students.

Dissemination

The purposes of Phase 2 of this project were:

- to disseminate information about the project design and results-to-date to faculty members in Developmental Studies and Mathematics at the two institutions, and
- to broaden the involvement of faculty from the two institutions in the implementation of the pilot data.

Phase 2 consisted of three dissemination activities:

1. Addition of two Mathematics faculty members (one from each institution) which resulted in the completion of a College Algebra Inventory.
2. Implementation of a faculty dissemination luncheon workshop for faculty at the two institutions.
3. Implementation of a faculty development retreat for Atlanta Metropolitan College.

College Algebra Inventory

Two additional mathematics faculty members were added to the project team--Dr. Frank Hall, Georgia State University and Dr. Joe Patterson, Atlanta Metropolitan College. In Phase 1 of this project, faculty members from the two institutions had analyzed the course syllabi, texts, and tests for the college algebra courses at the two institutions. Following this, they felt that an inventory of College Algebra operational skills or topics with a short description, an example, and sample problems of varying levels of difficulty would be useful to faculty endeavoring to help students move from developmental mathematics to regular mathematics courses at the two institutions and from one institution to another. A team of faculty from both Phase 1 and Phase 2 of the project produced this Inventory, obtaining feedback from other faculty members at both institutions during its development. Copies have been distributed to the faculty in both institutions (see Appendix E), and it is being prepared for national dissemination through one of the mathematics professional organizations.

Dissemination Conference

On February 16, 1993, a luncheon workshop was held for 50 faculty and administrators from Atlanta Metropolitan College and Georgia State University. The primary purpose was for project participants to describe the project and to present preliminary results to their colleagues in both Developmental Studies and Mathematics. Another purpose was for these faculties to become acquainted with one another. The highlight of the luncheon was a presentation by Dr. William Haver, Professor of Mathematical Sciences at Virginia Commonwealth University in Richmond. He was a participant in their National Center for Academic Achievement and Transfer Partnership Project with J. Sargeant Reynolds Community College and Virginia Commonwealth University. He shared their experiences in faculty exchanges, developing and team teaching a common precalculus mathematics course, and in joint presentations at professional meetings. An agenda for the workshop is included in Appendix F.

Curriculum Revision Workshop

The faculty at Atlanta Metropolitan College chose to use their faculty released time funds for a faculty development retreat which was held on December 10 & 11, 1992. The keynote speaker was Professor Harvey Brightman, Regents' Professor of Decision Sciences at Georgia State University. The purpose of the workshop was to provide a theoretical framework for the review and enhancement of course syllabi and to initiate the revision of course objectives to reflect institutional priorities (see Appendix G).

Disemination Products

In addition to the publication of the College Algebra Inventory and the dissemination conference, two workshops were presented at professional conferences (see Appendix H).

Conclusion

The overall goal of the partnership project was to increase articulation between developmental and regular mathematics courses within and between Georgia State

University and Atlanta Metropolitan College through examining the curriculum and instruction for mathematics coursed through precalculus at each institution. The study was conducted in three phases. Phase 1 was largely a pilot study involving a mathematical content analysis and the development and field-testing of faculty and student survey and classroom observation instruments. Phase 2 involved primarily the dissemination of Phase 1 developments. In Phase 3 a mathematical content analysis was conducted of the precalculus courses at each institution while a mathematical task analysis of instruction was performed using the revised instruments from Phase 1 to gather data.

It was hoped that such an examination would reveal significant factors which contribute to the number of minority transfer students from AMC enrolling in higher-level mathematics courses at GSU by improving the articulation of the mathematics courses at the two institutions. In the following paragraphs, the conclusions regarding the content and instruction will be delineated.

Content

With respect to the sequencing of mathematics courses at each institution, there are some differences which may pose possible complications for students transferring at specific points in the sequence. Specifically, there are three courses in the developmental mathematics sequence and one college algebra course at GSU, while there are two developmental mathematics courses, one college algebra course, and one transitional college algebra/precalculus course at AMC. Students transferring within a sequence would find either that material would be repeated or that some topics would not be covered because they would be assumed to have been covered in prerequisite courses. These difficulties could be avoided if students transferred at the end of a sequence rather than within a sequence.

The other issue appears to be the breadth of coverage. Fewer topics are covered in the college algebra and precalculus courses taught at AMC than in the comparable courses at GSU. Omission of additional topics important to success in subsequent courses could have a direct effect on the mathematics knowledge and understanding that transferring students have when entering GSU and on their likelihood of success in precalculus at GSU.

Instruction

The instructional practices employed at both institutions are quite similar. The slight variations noted for specific categories did not significantly change the overall similarities in the trends noted. Similar instructional methods included lecture, examples (either student or instructor), and student solutions. Although the average number of problems assigned varied greatly, classroom coverage of homework was similar. Faculty reported using the textbook as their primary resource.

As reported in the results and noted in the discussion following Phase 3, faculty and students varied in their perceptions and expectations about certain aspects of classroom, student preparation, and student expectations. Over the period of just one academic quarter, faculty revised their individual responses, in some cases completely. Providing faculty with these survey results may help them assess their instructional demands and practice by making them aware of and examining their own response profile.

The difference in the levels of classroom interaction noted through observations may indicate that learning styles rather than instructional practice may contribute more signicantly to the success rates in mathematics for transferring students. In addition to the difference in the demands of course content, transferring students may face a more analytical rather than relational learning environment requiring more analytic approaches to achieve success. For example, observational field notes suggested that GSU students were more passive, less verbal, and interacted less with their instructors than did AMC students who asked more questions and were more involved with the classroom activities. Research suggests that certain ethnic/cultural groups such as African-Americans tend to be relational learners as opposed to analytical learners, the method of instruction which traditional mathematics instruction favors (Ginsburg, 1978; Hilliard, 1988; Stiff, 1990). Methods of instruction which support the pedagogical recommendations of the NCTM Standards (1989, 1991) account for differences in learning styles and hence provide effective instruction for a broader range of learners. Students at the college level, especially those at GSU, would benefit from experiencing such instructional practices.

There is much that can be learned from efforts such as the GSU/AMC Partnership Project. We view these efforts as only a beginning. The groundwork has been laid for systematically assessing course content and instructional demands and practice. Perhaps this model can be used to address similar concerns in other disciplines.

Significance of the Partnership Project

Perhaps the most significant finding of the study was the opportunity for faculty from the two institutions to become acquainted with one another. As they talked, analyzed the content of their courses, and shared observations, commonalities and discrepancies both within and between institutions became apparent. At AMC, this discussion has led to significant curriculum reform. Two faculty retreats have been held, and major faculty effort has been devoted to revising the mathematics curriculum. While it is too soon to have transfer results, it is hoped that articulation between the mathematics programs at the institutions has been improved. Certainly, a dialogue has begun.

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Appendix A

GSU/AMC Faculty Participating in Project

Atlanta Metropolitan College

Betty Hood, Developmental Studies

Zacharia Menare, Mathematics

John Morrell, Mathematics

Barbara Morgan, Division of Mathematics & Natural Sciences

Joe Patterson, Mathematics

Georgia State University

Jean Bevis, Mathematics

Frank Hall, Mathematics

Lynn Hart, Developmental Studies

Deborah Najee-ullah, Developmental Studies

Joanne R. Nurss, Center for the Study of Adult Literacy

Appendix B

Results of the Content Analysis of Developmental Studies Mathematics and College Algebra Courses

Table 1 Summary of Tables 2 - 5
Average (AVR) values for Text and Exam of each course.

Table codes: Depth of coverage based upon text and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.

Courses	096	099	101	104
Real Numbers				
Kinds (positive/negative, rational/irrational)	3 2	1 0	1 1	1 1
Arith. of signed numbers, additive inverses	3 2	1 1	1 1	1 1
Multiplicative inverses	3 2	1 1	1 1	1 1
Equivalent expressions	3 2	1 1	1 1	1 1
Laws and properties of + and ×	3 2	1 1	1 1	1 1
Laws of (integer) exponents	3 2	2 0	1 1	2 1
Absolute value (definition and properties)	2 1	0 0	1 1	2 2
Intervals	3 2	0 0	1 1	2 1
Union and intesection of sets	0 0	0 0	1 1	3 2
Extension to complex numbers	0 1	3 2	0 0	3 2

	096	099	101	104
Algebraic Expressions (AEs)				
Polynomials (defined)	2 2	1 0	1 1	2 1
Add and subtract AEs	3 2	1 1	1 1	2 1
Multiply polynomials	3 2	1 1	3 2	2 1
Factoring polynomials - common factor	3 2	1 0	2 1	2 2
Factoring polynomials - difference of squares	3 2	1 0	3 2	2 2
Factoring quadratics	3 2	1 0	3 2	2 2
Factoring trinomials by quadratic tech.	3 2	3 0	3 2	2 2
Fractional expressions - simplifying	3 2	1 0	3 2	2 2
Fractional expressions +, -, ×, ÷	3 2	1 2	3 2	2 2
Fractions of fractions	0 1	1 0	3 2	2 1
Radical notation	3 2	3 2	3 2	3 1
Rational exponents, definition, laws	2 1	3 0	2 2	3 1
Division of polynomials	3 2	1 0	3 2	2 1

	096	099	101	104
Equations and Inequalities in One Unknown				
General principles	3 2	1 1	2 1	2 1
Linear equations	3 2	2 1	2 1	3 2
Equations with fractional expressions	3 2	2 2	2 1	3 2
Quadratic equations - factoring	3 2	3 0	2 1	3 2
Quadratic equations - completing the square	0 0	3 0	2 1	3 2
Quadratic formula - discriminant	2 0	3 2	2 1	3 2
Complex roots of quadratics	0 0	3 2	2 1	3 2
Equations with radicals	3 2	3 2	2 1	3 2
Equations reducible to quadratics	3 2	3 0	2 1	3 2
Equations with absolute value	2 0	0 0	2 1	3 2
Linear inequalities	3 2	3 0	2 1	3 2
Inequalities with absolute value	0 0	0 0	2 1	3 2
Quadratic inequalities	0 0	0 0	2 1	3 2
Inequalities with fractional expressions	2 1	0 0	2 1	3 2

	096	099	101	104
Problem Solving, Word Problems				
General techniques	3 2	2 1	3 1	3 1
Ratio and Proportion	3 2	2 0	3 2	0 0
Percent, Interest	3 2	2 2	0 0	2 0
Counting techniques	2 1	0 0	0 0	0 0
Probability	2 1	0 0	0 0	0 0
Averages, means	2 2	0 0	0 0	0 0
Geometric formula (Area, etc.)	3 2	3 2	0 0	2 1
Geometry of lines and triangles	3 2	3 2	0 0	1 1
Speed, velocity	2 1	2 2	0 0	2 2
Scientific notation	3 2	0 0	0 0	1 0
Distance formula in the plane	0 0	0 0	2 2	3 2
Quadratic problems	2 2	3 0	2 2	3 2
Exponential/Log models	0 0	2 0	0 0	3 2
Systems of equations	3 2	3 2	2 0	3 2
Mixtures	0 0	2 2	0 0	0 2
Exponents	2 2	2 0	3 2	0 0
Roots	2 1	2 0	3 2	2 1
Variation	0 0	3 0	0 0	0 0

	096	099	101	104
Functions				
Ordered pairs	3 2	3 0	3 2	3 1
Relations, domain, range	2 1	3 0	1 1	3 2
Composition	0 0	3 2	0 0	3 2
Algebra of functions	2 0	3 0	0 0	3 2
Graphing points	3 2	3 1	3 2	3 2
Graphing relations	2 1	3 0	3 2	3 2
Graphing functions	2 2	3 1	0 0	3 2
Vertical line test	0 0	3 0	0 0	3 2
Shift of axes	0 0	0 0	0 0	2 0
Domain/range of functions	2 1	3 0	0 0	3 2
Inverses of relations/functions	0 0	3 0	0 0	3 2
Symmetry about y = x	0 0	3 0	0 0	3 0
One to One functions, horizontal line test	0 0	3 0	0 0	3 2
Formula for an inverse	0 0	3 0	0 0	3 2
Increasing/Decreasing functions	0 0	2 0	0 0	3 2
The difference quotient	0 0	0 0	0 0	3 1

	096	099	101	104
Equations and Inequalities in Two Variables				
Linear equations - graphing, slope	3 2	3 0	3 2	3 2
Lines - Point slope equation	2 1	3 0	3 2	3 2
Lines - Two point equation	2 1	3 0	3 2	3 2
Lines - Slope intercept equation	3 2	3 2	3 2	3 2
Parallel and perpendicular lines	2 2	3 0	3 2	3 2
Midpoints of line seqments	0 0	0 0	3 2	3 2
Equations of circles	0 0	0 0	0 0	3 2
Quadratic equations - graphing parabolas	2 2	3 2	3 2	3 2
Quadratic equations - intercepts	2 1	1 2	3 2	3 2
Quadratic equations - vertex, max/min	0 0	3 2	2 2	3 2
Quadratic equations - symmetry	0 0	3 0	3 2	3 1
Roots, Radicals and exponents	2 1	0 0	3 2	3 2

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Exponential and Log Functions	096	099	101	104
Graphs of exponential functions	0 1	3 0	0 2	3 2
Log functions as inverses of exp func	0 0	3 0	0 0	3 2
Graphs of log functions	0 0	3 0	0 0	3 2
Conversion: Log ↔ Exp	0 0	3 1	0 0	3 2
Properties of logs	0 0	3 1	0 1	3 2
Change of base	0 0	2 0	0 0	3 1
Natural logs	0 0	2 0	0 0	3 2
Solving equations	0 0	3 2	0 2	3 2

Systems of (mostlyLinear) Eqs/Ineqs, Matrices	096	099	101	104
Graphical solution	2 1	2 0	3 2	3 1
Substitution method	0 0	3 2	3 2	3 2
Elimination/Addition method	2 1	3 2	3 2	3 2
Systems with three variables	0 0	0 0	0 0	3 2
Consistent/Inconsistent systems	0 0	3 0	2 2	3 1
Independent/Dependent systems	0 0	3 0	2 2	3 1
Homogeneous systems	0 0	0 0	0 0	3 1
Augmented matrix of a system	0 0	0 0	0 0	3 2
Arithmetic operations on matrices	0 0	0 0	0 0	3 2
System as Ax = b	0 0	0 0	0 0	3 2
Inverses of matrices	0 0	0 0	0 0	3 0
Determinants	0 0	0 0	0 0	0 0
Cramers rule	0 0	0 0	0 0	0 0
Systems of inequalities	0 0	0 0	2 2	0 0

Polynomials	096	099	101	104
Graphs	0 0	1 0	0 0	2 0
Division algorithm	0 1	0 0	0 0	3 0
Roots and factors	0 0	0 0	3 2	3 0
Remainder theorem	0 0	0 0	2 0	3 0
Synthetic division	0 0	0 0	2 0	2 1
Fundamental theorem of algebra	0 0	0 0	3 0	3 0
Polynomials with given roots	0 0	1 0	2 0	2 0

Table 2. Topics for DS 96
(based on four text responses and three exam responses)

Table codes: Depth of coverage based upon text and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frequency of average

	Text			Exam		
Real Numbers	RNG	AVR	FOA	RNG	AVR	FOA
Kinds (pos./neg., rat./irrat.)	3-3	3	4	2-2	2	3
Arith. of signed numbers, add. invs.	2-3	3	3	1-2	2	2
Multiplicative inverses	2-3	3	3	1-2	2	2
Equivalent expressions	2-3	3	3	1-2	2	2
Laws and properties of + and ×	2-3	3	3	1-2	2	2
Laws of (integer) exponents	3-3	3	4	2-2	2	3
Absolute value (def. and properties)	2-2	2	4	1-1	1	3
Intervals	2-3	3	2	1-2	2	2
Union and intesection of sets	0-2	0	3	0-1	0	2
Extension to complex numbers	0-2	0	3	0-2	1	1

	Text			Exam		
Algebraic Expressions (AEs)	RNG	AVR	FOA	RNG	AVR	FOA
Polynomials (defined)	0-3	2	1	1-2	2	2
Add and subtract AEs	2-3	3	3	1-2	2	2
Multiply polynomials	3-3	3	4	2-2	2	3
Factoring polynomials - common factor	3-3	3	4	2-2	2	3
Factoring polynomials - diff. of squares	2-3	3	3	2-2	2	3
Factoring quadratics	2-3	3	3	2-2	2	3
Factoring trinomials by quadratic tech.	2-3	3	3	2-2	2	3
Fractional expressions - simplifying	3-3	3	4	2-2	2	3
Fractional expressions +, -, ×, ÷	3-3	3	4	2-2	2	3
Fractions of fractions	0-2	0	3	0-2	1	0
Radical notation	2-3	3	3	2-2	2	3
Rational exponents, definition, laws	0-3	2	2	0-2	1	1
Division of polynomials	0-3	3	3	2-2	2	3

	Text			Exam		
Equations and Ineq. in One Unknown	RNG	AVR	FOA	RNG	AVR	FOA
General principles	3-3	3	4	2-2	2	3
Linear equations	3-3	3	3	2-2	2	3
Equations with fractional expressions	3-3	3	4	2-2	2	3
Quadratic equations - factoring	3-3	3	4	2-2	2	3
Quadratic equations - compl. the square	0-3	0	3	0-0	0	3
Quadratic formula - discriminant	0-3	2	2	0-1	0	2
Complex roots of quadratics	0-2	0	3	0-1	0	2
Equations with radicals	2-3	3	3	2-2	2	3
Equations reducible to quadratics	0-3	3	3	2-2	2	3
Equations with absolute value	0-2	2	2	0-1	0	2
Linear inequalities	2-3	3	3	2-2	2	3
Inequalities with absolute value	0-0	0	4	0-0	0	3
Quadratic inequalities	0-2	0	3	0-1	0	2
Inequalities with fractional expressions	0-3	2	1	1-2	1	1

	Text			Exam		
Problem Solving, Word Problems	RNG	AVR	FOA	RNG	AVR	FOA
General techniques	3-3	3	4	1-2	2	2
Ratio and Proportion	2-3	3	3	2-2	2	3
Percent, Interest	2-3	3	3	1-2	2	2
Counting techniques	0-2	2	2	0-2	1	0
Probability	0-3	2	2	0-2	1	1
Averages, means	2-3	2	3	1-2	2	2
Geometric formula (Area, etc.)	2-3	3	3	2-2	2	3
Geometry of lines and triangles	2-3	3	3	2-2	2	3
Speed, velocity	0-2	2	2	0-2	1	1
Scientific notation	2-3	3	2	2-2	2	3
Distance formula in the plane	0-2	0	3	0-1	0	2
Quadratic problems	0-3	2	1	1-2	2	2
Exponential/Log models	0-2	0	3	0-1	0	2
Systems of equations	2-3	3	3	1-2	2	2
Mixtures	0-2	0	3	0-1	0	2
Exponents	0-3	2	1	2-2	2	3
Roots	0-3	2	0	0-2	1	1
Variation	0-2	0	3	0-0	0	3

	Text			Exam		
Functions	RNG	AVR	FOA	RNG	AVR	FOA
Ordered pairs	2-3	3	3	2-2	2	3
Relations, domain, range	0-3	2	2	0-2	1	1
Composition	0-0	0	4	0-0	0	3
Algebra of functions	0-2	2	2	0-1	0	2
Graphing points	2-3	3	3	2-2	2	3
Graphing relations	0-2	2	2	1-2	1	2
Graphing functions	0-3	2	2	1-2	2	2
Vertical line test	0-0	0	4	0-0	0	3
Shift of axes	0-0	0	4	0-0	0	3
Domain/range of functions	0-3	2	2	0-2	1	1
Inverses of relations/functions	0-0	0	4	0-0	0	3
Symmetry about y = x	0-0	0	4	0-0	0	3
1-to-1 functions, horizontal line test	0-0	0	4	0-0	0	3
Formula for an inverse	0-0	0	4	0-0	0	3
Increasing/Decreasing functions	0-0	0	4	0-0	0	3
The difference quotient	0-0	0	4	0-0	0	3

	Text			Exam		
Equations and Ineqs. in Two Variables	RNG	AVR	FOA	RNG	AVR	FOA
Linear equations - graphing, slope	2-3	3	3	2-2	2	3
Lines - Point slope equation	0-3	2	2	0-2	1	0
Lines - Two point equation	0-2	2	2	0-2	1	0
Lines - Slope intercept equation	2-3	3	3	2-2	2	3
Parallel and perpendicular lines	0-3	2	2	1-2	2	2
Midpoints of line seqments	0-0	0	4	0-0	0	3
Equations of circles	0-0	0	4	0-0	0	3
Quad. equations - graphing parabolas	0-2	2	3	1-2	2	2
Quadratic equations - intercepts	0-3	2	2	0-2	1	0
Quadratic equations - vertex, max/min	0-2	0	3	0-1	0	2
Quadratic equations - symmetry	0-0	0	4	0-1	0	2
Roots, Radicals and exponents	0-3	2	0	0-2	1	0

	Text			Exam		
Exponential and Log Functions	RNG	AVR	FOA	RNG	AVR	FOA
Graphs of exponential functions	0-2	0	3	0-2	1	0
Log functions as inverses of exp func	0-0	0	4	0-0	0	3
Graphs of log functions	0-0	0	4	0-0	0	3
Conversion: Log ↔ Exp	0-0	0	4	0-0	0	3
Properties of logs	0-0	0	4	0-0	0	3
Change of base	0-0	0	4	0-0	0	3
Natural logs	0-0	0	4	0-0	0	3
Solving equations	0-0	0	4	0-0	0	4

	Text			Exam		
Systems (mostly Lin.) Eqs/Ineqs, Matrices	RNG	AVR	FOA	RNG	AVR	FOA
Graphical solution	0-3	2	1	0-2	1	0
Substitution method	0-2	0	3	0-0	0	3
Elimination/Addition method	0-3	2	1	0-2	1	0
Systems with three variables	0-0	0	4	0-0	0	3
Consistent/Inconsistent systems	0-2	0	3	0-0	0	3
Independent/Dependent systems	0-0	0	4	0-0	0	3
Homogeneous systems	0-0	0	4	0-0	0	3
Augmented matrix of a system	0-0	0	4	0-0	0	3
Arithmetic operations on matrices	0-0	0	4	0-0	0	3
System as Ax = b	0-0	0	4	0-0	0	3
Inverses of matrices	0-0	0	4	0-0	0	3
Determinants	0-0	0	4	0-0	0	3
Cramers rule	0-0	0	4	0-0	0	3
Systems of inequalities	0-0	0	4	0-0	0	3

	Text			Exam		
Polynomials	RNG	AVR	FOA	RNG	AVR	FOA
Graphs	0-0	0	4	0-1	0	2
Division algorithm	0-2	0	2	1-2	1	1
Roots and factors	0-0	0	4	0-0	0	3
Remainder theorem	0-0	0	4	0-0	0	3
Synthetic division	0-0	0	4	0-0	0	3
Fundamental theorem of algebra	0-0	0	4	0-0	0	3
Polynomials with given roots	0-0	0	4	0-0	0	3

Table 3 Topics for DS 99
(based on three responses on text, and one departmental final)

Table codes: Depth of coverage based upon text and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frequency of average

	Text			Exam		
Real Numbers	RNG	AVR	FOA	RNG	AVR	FOA
Kinds (pos./neg., rat./irrat.)	1-2	1	2		0	
Arith. of signed numbers, add. invs.	1-2	1	2		1	
Multiplicative inverses	1-2	1	2		1	
Equivalent expressions	1-2	1	2		1	
Laws and properties of + and ×	1-2	1	2		1	
Laws of (integer) exponents	1-3	2	1		0	
Absolute value (def. and properties)	0-0	0	3		0	
Intervals	0-2	0	2		0	
Union and intesection of sets	0-0	0	3		0	
Extension to complex numbers	2-3	3	2		2	

	Text			Exam		
Algebraic Expressions (AEs)	RNG	AVR	FOA	RNG	AVR	FOA
Polynomials (defined)	1-1	1	3		0	
Add and subtract AEs	1-2	1	2		1	
Multiply polynomials	1-2	1	2		1	
Factoring polynomials - common factor	1-2	1	2		0	
Factoring polynomials - diff. of squares	1-2	1	2		0	
Factoring quadratics	0-3	1	1		0	
Factoring trinomials by quadratic tech.	0-3	3	2		0	
Fractional expressions - simplifying	0-1	1	2		0	
Fractional expressions +, -, ×, +	0-1	1	2		2	
Fractions of fractions	0-2	1	1		0	
Radical notation	3-3	3	3		2	
Rational exponents, definition, laws	2-3	3	2		0	
Division of polynomials	0-1	1	2		0	

	Text			Exam		
Equations and Ineq. in One Unknown	RNG	AVR	FOA	RNG	AVR	FOA
General principles	0-3	1	1		1	
Linear equations	1-3	2	1		1	
Equations with fractional expressions	1-3	2	1		2	
Quadratic equations - factoring	2-3	3	2		0	
Quadratic equations - compl. the square	2-3	3	2		0	
Quadratic formula - discriminant	3-3	3	3		2	
Complex roots of quadratics	3-3	3	3		2	
Equations with radicals	3-3	3	3		2	
Equations reducible to quadratics	2-3	3	2		0	
Equations with absolute value	0-0	0	3		0	
Linear inequalities	0-3	3	2		0	
Inequalities with absolute value	0-0	0	3		0	
Quadratic inequalities	0-3	0	2		0	
Inequalities with fractional expressions	0-3	0	2		0	

	Text			Exam		
Problem Solving, Word Problems	RNG	AVR	FOA	RNG	AVR	FOA
General techniques	2-3	2	2		1	
Ratio and Proportion	0-2	2	2		0	
Percent, Interest	2-3	2	2		2	
Counting techniques	0-3	0	2		0	
Probability	0-0	0	3		0	
Averages, means	0-0	0	3		0	
Geometric formula (Area, etc.)	2-3	3	2		2	
Geometry of lines and triangles	2-3	3	2		2	
Speed, velocity	0-3	2	1		2	
Scientific notation	0-2	0	2		0	
Distance formula in the plane	0-3	0	2		0	
Quadratic problems	3-3	3	3		0	
Exponential/Log models	0-3	2	1		0	
Systems of equations	3-3	3	3		2	
Mixtures	0-3	2	1		2	
Exponents	0-3	2	1		0	
Roots	0-3	2	1		0	
Variation	2-3	3	2		0	

	Text			Exam		
Functions	RNG	AVR	FOA	RNG	AVR	FOA
Ordered pairs	2-3	3	2		0	
Relations, domain, range	2-3	3	2		0	
Composition	2-3	3	2		2	
Algebra of functions	2-3	3	2		0	
Graphing points	2-3	3	2		1	
Graphing relations	2-3	3	2		0	
Graphing functions	2-3	3	2		1	
Vertical line test	2-3	3	2		0	
Shift of axes	0-2	0	2		0	
Domain/range of functions	2-3	3	2		0	
Inverses of relations/functions	3-3	3	3		0	
Symmetry about y = x	2-3	3	2		0	
1-to-1 functions, horizontal line test	3-3	3	3		0	
Formula for an inverse	3-3	3	3		0	
Increasing/Decreasing functions	0-2	2	2		0	
The difference quotient	0-2	0	2		0	

	Text			Exam		
Equations and Ineqs. in Two Variables	RNG	AVR	FOA	RNG	AVR	FOA
Linear equations - graphing, slope	1-3	3	2		0	
Lines - Point slope equation	1-3	3	2		0	
Lines - Two point equation	1-3	3	2		0	
Lines - Slope intercept equation	1-3	3	2		2	
Parallel and perpendicular lines	2-3	3	2		0	
Midpoints of line seqments	0-3	0	2		0	
Equations of circles	0-2	0	2		0	
Quad. equations - graphing parabolas	3-3	3	3		2	
Quadratic equations - intercepts	0-3	1	1		2	
Quadratic equations - vertex, max/min	2-3	3	2		2	
Quadratic equations - symmetry	0-3	3	2		0	
Roots, Radicals and exponents	0-3	0	2		0	

	Text			Exam		
Exponential and Log Functions	RNG	AVR	FOA	RNG	AVR	FOA
Graphs of exponential functions	2-3	3	2		0	
Log functions as inverses of exp func	2-3	3	2		0	
Graphs of log functions	2-3	3	2		0	
Conversion: Log ↔ Exp	3-3	3	3		1	
Properties of logs	3-3	3	3		1	
Change of base	2-3	2	2		0	
Natural logs	2-3	2	2		0	
Solving equations	3-3	3	3		2	

	Text			Exam		
Systems (mostly Lin.) Eqs/Ineqs, Matrices	RNG	AVR	FOA	RNG	AVR	FOA
Graphical solution	0-3	2	1		0	
Substitution method	0-3	3	2		2	
Elimination/Addition method	0-3	3	2		2	
Systems with three variables	0-2	0	2		0	
Consistent/Inconsistent systems	2-3	3	2		0	
Independent/Dependent systems	2-3	3	2		0	
Homogeneous systems	0-3	0	2		0	
Augmented matrix of a system	0-0	0	3		0	
Arithmetic operations on matrices	0-0	0	3		0	
System as Ax = b	0-2	0	2		0	
Inverses of matrices	0-0	0	3		0	
Determinants	0-2	0	2		0	
Cramers rule	0-0	0	3		0	
Systems of inequalities	0-0	0	3		0	

	Text			Exam		
Polynomials	RNG	AVR	FOA	RNG	AVR	FOA
Graphs	0-2	1	1		0	
Division algorithm	0-0	0	3		0	
Roots and factors	0-1	0	2		0	
Remainder theorem	0-0	0	3		0	
Synthetic division	0-0	0	3		0	
Fundamental theorem of algebra	0-0	0	3		0	
Polynomials with given roots	0-2	1	1		0	

Table 4 Topics for Math 101
(based on 8 responses)

Table codes: Depth of coverage based upon text and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frequency of average

	Text			Exam		
Real Numbers	RNG	AVR	FOA	RNG	AVR	FOA
Kinds (pos./neg., rat./irrat.)	1-3	1	5	1-2	1	7
Arith. of signed numbers, add. invs.	1-3	1	5	1-2	1	7
Multiplicative inverses	1-2	1	5	1-1	1	8
Equivalent expressions	1-2	1	5	1-1	1	8
Laws and properties of + and ×	1-3	1	5	1-2	1	7
Laws of (integer) exponents	1-3	1	5	1-2	1	7
Absolute value (def. and properties)	1-3	1	5	1-2	1	7
Intervals	1-2	1	6	0-1	1	7
Union and intesection of sets	1-1	1	8	0-1	1	7
Extension to complex numbers	0-1	0	7	0-0	0	8

	Text			Exam		
Algebraic Expressions (AEs)	RNG	AVR	FOA	RNG	AVR	FOA
Polynomials (defined)	1-3	1	5	1-2	1	7
Add and subtract AEs	1-3	1	5	1-2	1	6
Multiply polynomials	3-3	3	8	2-2	2	8
Factoring polynomials - common factor	2-3	2	5	1-2	1	6
Factoring polynomials - diff. of squares	3-3	3	8	2-2	2	8
Factoring quadratics	3-3	3	8	2-2	2	8
Factoring trinomials by quadratic tech.	3-3	3	8	2-2	2	8
Fractional expressions - simplifying	3-3	3	8	2-2	2	8
Fractional expressions +, -, ×, ÷	3-3	3	8	2-2	2	8
Fractions of fractions	2-3	3	7	1-2	2	7
Radical notation	3-3	3	8	2-2	2	8
Rational exponents, definition, laws	2-3	2	5	2-2	2	8
Division of polynomials	3-3	3	8	2-2	2	8

	Text			Exam		
Equations and Ineq. in One Unknown	RNG	AVR	FOA	RNG	AVR	FOA
General principles	2-3	2	7	1-2	1	7
Linear equations	2-3	2	6	1-2	1	6
Equations with fractional expressions	2-3	2	6	1-2	1	6
Quadratic equations - factoring	2-3	2	6	1-2	1	6
Quadratic equations - compl. the square	2-3	2	6	1-2	1	6
Quadratic formula - discriminant	2-3	2	6	1-2	1	6
Complex roots of quadratics	2-3	2	7	1-2	1	7
Equations with radicals	2-3	2	6	1-2	1	7
Equations reducible to quadratics	2-3	2	6	1-2	1	7
Equations with absolute value	2-3	2	6	1-2	1	7
Linear inequalities	2-3	2	7	1-1	1	8
Inequalities with absolute value	2-3	2	7	1-1	1	8
Quadratic inequalities	0-3	2	5	0-1	1	7
Inequalities with fractional expressions	0-2	2	7	0-1	1	7

	Text			Exam		
Problem Solving, Word Problems	RNG	AVR	FOA	RNG	AVR	FOA
General techniques	2-3	3	7	1-1	1	8
Ratio and Proportion	3-3	3	8	2-2	2	8
Percent, Interest	0-2	0	7	0-0	0	8
Counting techniques	0-0	0	8	0-0	0	8
Probability	0-0	0	8	0-0	0	8
Averages, means	0-0	0	8	0-0	0	8
Geometric formula (Area, etc.)	0-2	0	6	0-2	0	7
Geometry of lines and triangles	0-2	0	7	0-0	0	8
Speed, velocity	0-2	0	6	0-0	0	8
Scientific notation	0-3	0	4	0-2	0	6
Distance formula in the plane	0-2	2	7	0-2	2	7
Quadratic problems	2-2	2	8	1-2	2	7
Exponential/Log models	0-0	0	8	0-0	0	8
Systems of equations	2-3	2	6	0-2	0	6
Mixtures	0-0	0	8	0-0	0	8
Exponents	3-3	3	8	2-2	2	8
Roots	3-3	3	8	2-2	2	8
Variation	0-0	0	8	0-0	0	8

	Text			Exam		
Functions	RNG	AVR	FOA	RNG	AVR	FOA
Ordered pairs	3-3	3	8	2-2	2	8
Relations, domain, range	1-3	1	7	1-2	1	6
Composition	0-0	0	8	0-0	0	8
Algebra of functions	0-2	0	7	0-1	0	7
Graphing points	3-3	3	8	2-2	2	8
Graphing relations	2-3	3	7	1-2	2	7
Graphing functions	0-0	0	8	0-0	0	8
Vertical line test	0-0	0	8	0-0	0	8
Shift of axes	0-0	0	8	0-0	0	8
Domain/range of functions	0-0	0	8	0-0	0	8
Inverses of relations/functions	0-0	0	8	0-0	0	8
Symmetry about y = x	0-0	0	8	0-0	0	8
1-to-1 functions, horizontal line test	0-0	0	8	0-0	0	8
Formula for an inverse	0-0	0	8	0-0	0	8
Increasing/Decreasing functions	0-0	0	8	0-0	0	8
The difference quotient	0-0	0	8	0-0	0	8

	Text			Exam		
Equations and Ineqs. in Two Variables	RNG	AVR	FOA	RNG	AVR	FOA
Linear equations - graphing, slope	3-3	3	8	2-2	2	8
Lines - Point slope equation	3-3	3	8	2-2	2	8
Lines - Two point equation	3-3	3	8	2-2	2	8
Lines - Slope intercept equation	3-3	3	8	2-2	2	8
Parallel and perpendicular lines	2-3	3	7	1-2	2	7
Midpoints of line seqments	0-3	3	6	0-2	2	7
Equations of circles	0-0	0	8	0-0	0	8
Quad. equations - graphing parabolas	0-3	3	4	0-2	2	6
Quadratic equations - intercepts	0-3	3	6	0-2	2	6
Quadratic equations - vertex, max/min	0-3	2	2	0-2	2	6
Quadratic equations - symmetry	0-3	3	4	0-2	2	6
Roots, Radicals and exponents	3-3	3	8	2-2	2	8

	Text			Exam		
Exponential and Log Functions	RNG	AVR	FOA	RNG	AVR	FOA
Graphs of exponential functions	0-3	0	6	0-2	2	6
Log functions as inverses of exp func	0-0	0	8	0-0	0	8
Graphs of log functions	0-3	0	6	0-2	0	6
Conversion: Log ↔ Exp	0-0	0	8	0-0	0	8
Properties of logs	0-2	0	6	0-1	1	6
Change of base	0-0	0	8	0-0	0	8
Natural logs	0-0	0	8	0-0	0	8
Solving equations	0-3	0	5	0-2	2	7

	Text			Exam		
Systems (mostly Lin.) Eqs/Ineqs, Matrices	RNG	AVR	FOA	RNG	AVR	FOA
Graphical solution	3-3	3	8	2-2	2	8
Substitution method	3-3	3	8	2-2	2	8
Elimination/Addition method	3-3	3	8	2-2	2	8
Systems with three variables	0-0	0	8	0-0	0	8
Consistent/Inconsistent systems	0-3	2	6	0-2	2	6
Independent/Dependent systems	0-3	2	6	0-2	2	6
Homogeneous systems	0-0	0	8	0-0	0	8
Augmented matrix of a system	0-0	0	8	0-0	0	8
Arithmetic operations on matrices	0-0	0	8	0-0	0	8
System as Ax = b	0-0	0	8	0-0	0	8
Inverses of matrices	0-0	0	8	0-0	0	8
Determinants	0-0	0	8	0-0	0	8
Cramer's rule	0-0	0	8	0-0	0	8
Systems of inequalities	0-2	2	5	0-2	2	6

	Text			Exam		
Polynomials	RNG	AVR	FOA	RNG	AVR	FOA
Graphs	0-2	0	6	0-0	0	8
Division algorithm	0-3	0	6	0-2	0	7
Roots and factors	0-3	3	6	2-2	2	8
Remainder theorem	0-3	2	6	0-2	0	7
Synthetic division	0-3	2	6	0-2	0	7
Fundamental theorem of algebra	0-3	3	6	0-2	0	6
Polynomials with given roots	0-3	2	5	0-2	0	7

Table 5 Topics for Math 104
(based on seven responses)

Table codes: Depth of coverage based upon text and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frequency of average

	Text			Exam		
Real Numbers	RNG	AVR	FOA	RNG	AVR	FOA
Kinds (pos./neg., rat./irrat.)	1-2	1	4	1-1	1	7
Arith. of signed numbers, add. invs.	1-2	1	6	1-1	1	7
Multiplicative inverses	1-2	1	6	1-1	1	7
Equivalent expressions	0-2	1	3	1-1	1	7
Laws and properties of + and ×	1-2	1	4	1-1	1	7
Laws of (integer) exponents	1-3	2	5	1-2	1	6
Absolute value (def. and properties)	2-3	2	4	1-2	2	4
Intervals	2-3	2	4	1-2	1	4
Union and intesection of sets	0-3	3	6	2-2	2	7
Extension to complex numbers	2-3	3	6	2-2	2	7

	Text			Exam		
Algebraic Expressions (AEs)	RNG	AVR	FOA	RNG	AVR	FOA
Polynomials (defined)	1-3	2	4	1-1	1	7
Add and subtract AEs	1-2	2	5	1-2	1	6
Multiply polynomials	1-3	2	4	1-2	1	4
Factoring polynomials - common factor	1-3	2	3	1-2	2	4
Factoring polynomials - diff. of squares	1-3	2	3	1-2	2	4
Factoring quadratics	1-3	2	2	1-2	2	5
Factoring trinomials by quadratic tech.	1-3	2	2	1-2	2	4
Fractional expressions - simplifying	1-3	2	3	1-2	2	5
Fractional expressions +, -, ×, ÷	1-3	2	3	1-2	2	4
Fractions of fractions	1-3	2	4	0-2	1	3
Radical notation	1-3	3	4	1-2	1	4
Rational exponents, definition, laws	1-3	3	4	0-2	1	3
Division of polynomials	0-3	2	3	0-2	1	3

	Text			Exam		
Equations and Ineq. in One Unknown	RNG	AVR	FOA	RNG	AVR	FOA
General principles	2-3	2	4	1-2	1	5
Linear equations	2-3	3	5	1-2	2	6
Equations with fractional expressions	3-3	3	7	2-2	2	7
Quadratic equations - factoring	3-3	3	7	1-2	2	6
Quadratic equations - compl. the square	3-3	3	7	0-2	2	6
Quadratic formula - discriminant	3-3	3	7	2-2	2	7
Complex roots of quadratics	3-3	3	7	1-2	2	6
Equations with radicals	3-3	3	7	2-2	2	7
Equations reducible to quadratics	3-	3	7	0-2	2	6
Equations with absolute value	3-3	3	7	0-2	2	6
Linear inequalities	3-3	3	7	2-2	2	7
Inequalities with absolute value	3-3	3	7	2-2	2	7
Quadratic inequalities	3-3	3	7	2-2	2	7
Inequalities with fractional expressions	3-3	3	7	0-2	2	5

	Text			Exam		
Problem Solving, Word Problems	RNG	AVR	FOA	RNG	AVR	FOA
General techniques	2-3	3	5	1-2	1	5
Ratio and Proportion	0-2	0	5	0-1	0	6
Percent, Interest	0-3	2	3	0-2	0	5
Counting techniques	0-0	0	7	0-0	0	7
Probability	0-0	0	7	0-0	0	7
Averages, means	0-2	0	4	0-1	0	5
Geometric formula (Area etc.)	0-3	2	2	0-2	1	4
Geometry of lines and triangles	0-3	1	2	0-2	1	2
Speed, velocity	0-3	2	1	0-2	2	4
Scientific notation	0-2	1	2	0-2	0	4
Distance formula in the plane	2-3	3	6	0-2	2	4
Quadratic problems	2-3	3	6	0-2	2	5
Exponential/Log models	0-3	3	6	0-2	2	5
Systems of equations	1-3	3	6	0-2	2	6
Mixtures	0-3	0	5	0-2	2	6
Exponents	0-3	0	4	0-2	0	4
Roots	0-3	2	1	0-2	1	1
Variation	0-0	0	7	0-0	0	7

	Text			Exam		
Functions	RNG	AVR	FOA	RNG	AVR	FOA
Ordered pairs	3-3	3	7	1-2	1	4
Relations, domain, range	3-3	3	7	0-2	2	5
Composition	3-3	3	7	1-2	2	6
Algebra of functions	3-3	3	7	0-2	2	4
Graphing points	1-3	3	6	1-2	2	4
Graphing relations	3-3	3	7	0-2	2	4
Graphing functions	3-3	3	7	1-2	2	6
Vertical line test	2-3	3	6	0-2	2	5
Shift of axes	0-3	2	2	0-1	0	6
Domain/range of functions	3-3	3	7	0-2	2	5
Inverses of relations/functions	3-3	3	7	0-2	2	6
Symmetry about y = x	0-3	3	5	0-2	0	4
1-to-1 functions, horizontal line test	2-3	3	6	0-2	2	4
Formula for an inverse	3-3	3	7	0-2	2	6
Increasing/Decreasing functions	3-3	3	7	0-2	2	5
The difference quotient	0-3	3	4	0-2	1	1

	Text			Exam		
Equations and Ineqs. in Two Variables	RNG	AVR	FOA	RNG	AVR	FOA
Linear equations - graphing, slope	3-3	3	7	0-2	2	6
Lines - Point slope equation	3-3	3	7	2-2	2	7
Lines - Two point equation	3-3	3	7	0-2	2	5
Lines - Slope intercept equation	3-3	3	7	0-2	2	5
Parallel and perpendicular lines	3-3	3	7	2-2	2	7
Midpoints of line seqments	3-3	3	7	0-2	2	6
Equations of circles	0-3	3	6	0-2	2	5
Quad. equations - graphing parabolas	3-3	3	7	2-2	2	7
Quadratic equations - intercepts	3-3	3	7	2-2	2	7
Quadratic equations - vertex, max/min	3-3	3	7	2-2	2	7
Quadratic equations - symmetry	0-3	3	5	0-2	1	1
Roots, Radicals and exponents	0-3	3	5	0-2	2	4

	Text			Exam		
Exponential and Log Functions	RNG	AVR	FOA	RNG	AVR	FOA
Graphs of exponential functions	3-3	3	7	0-2	2	4
Log functions as inverses of exp func	3-3	3	7	0-2	2	5
Graphs of log functions	3-3	3	7	0-2	2	4
Conversion: Log ↔ Exp	3-3	3	7	1-2	2	6
Properties of logs	3-3	3	7	2-2	2	7
Change of base	0-3	3	5	0-2	1	1
Natural logs	3-3	3	7	0-2	2	6
Solving equations	3-3	3	7	2-2	2	7

	Text			Exam		
Systems (mostly Lin.) Eqs/Ineqs, Matrices	RNG	AVR	FOA	RNG	AVR	FOA
Graphical solution	2-3	3	6	0-2	1	1
Substitution method	3-3	3	7	0-2	2	5
Elimination/Addition method	3-3	3	7	0-2	2	6
Systems with three variables	3-3	3	7	0-2	2	6
Consistent/Inconsistent systems	3-3	3	7	0-2	1	3
Independent/Dependent systems	3-3	3	7	0-2	1	3
Homogeneous systems	0-3	3	6	0-2	1	2
Augmented matrix of a system	2-3	3	6	0-2	2	4
Arithmetic operations on matrices	3-3	3	7	2-2	2	7
System as $Ax = b$	0-3	3	5	0-2	2	5
Inverses of matrices	0-3	3	4	0-2	0	4
Determinants	0-3	0	6	0-0	0	7
Cramers rule	0-0	0	7	0-0	0	7
Systems of inequalities	0-0	0	7	0-0	0	7

	Text			Exam		
Polynomials	RNG	AVR	FOA	RNG	AVR	FOA
Graphs	0-3	2	1	0-2	0	4
Division algorithm	0-3	3	4	0-2	0	4
Roots and factors	0-3	3	4	0-2	0	4
Remainder theorem	0-3	3	4	0-2	0	4
Synthetic division	0-3	2	1	0-2	1	1
Fundamental theorem of algebra	0-3	3	4	0-2	0	4
Polynomials with given roots	0-3	2	1	0-2	0	4

Appendix C

Results of the Content Analysis of Precalculus Courses

Table 1 Topics for Math 102 AMC
(based on departmental sylabus and five final examinations)

Table codes: Depth of coverage based upon test and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered.
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frequency of average.

	Text	Exam		
Real Numbers	AVR	RNG	AVR	FOA
Arith. of signed numbers, additive inverses	1	1-1	1	5
Multiplicative inverses	1	0-0	0	5
Equivalent expressions	1	1-1	1	5
Properties of + and ×	1	1-1	1	5
Integer exponents: notation, laws	1	1-2	1	4
Absolute value (definition and properties)	1	0-0	0	5
Intervals	1	0-0	0	5
Union and intesection of sets	1	0-0	0	5
Arithmetic of complex numbers	3	0-2	0	3

	Text	Exam		
Algebraic Expressions (AEs)	AVR	RNG	AVR	FOA
Polynomials (definition, recognition)	1	1-1	1	5
Addition and subtraction of AEs	1	1-1	1	5
Multiplying polynomials	1	0-1	1	3
Factoring polynomials - common factor	1	0-1	1	4
Factoring polynomials - difference of squares	1	0-0	0	5
Factoring quadratics	1	0-1	1	3
Factoring by quadratic technique	1	0-0	0	5
Factoring special cubic polynomials	1	0-2	0	4
Fractional expressions - simplifying	1	0-1	0	4
Fractional expressions +, -, ×, ÷	1	0-0	0	5
Fractions of fractions -	1	0-0	0	5
Radical notation	1	0-0	0	5
Rational exponents, definition, laws	1	0-2	0	4
Division of polynomials	0	0-0	0	5

	Text	Exam		
Polynomials	AVR	RNG	AVR	FOA
Graphs	1	1-1	1	5
Division algorithm	0	0-0	0	5
Roots and factors	1	0-2	0	4
Remainder theorem	0	0-0	0	5
Synthetic division	0	0-0	0	5
Fundamental theorem of algebra	0	0-0	0	5
Polynomials with given roots	0	0-0	0	5

	Text	Exam		
Equations and Inequalities in One Unknown	AVR	RNG	AVR	FOA
General principles	1	1-2	1	4
Linear equations	1	1-1	1	5
Quadratic equations - factoring	1	0-1	1	4
Quadratic equations - completing the square	1	0-1	1	3
Quadratic formula - discriminant	1	0-2	0	3
Complex roots of quadratics	2	0-0	0	5
Equations with radicals	3	0-2	0	3
Equations reducible to quadratics	0	0-0	0	5
Equations with fractional expressions	1	0-2	0	4
Equations with absolute value	2	0-0	0	5
Linear inequalities	3	0-0	0	5
Inequalities with absolute value	3	0-2	0	3
Quadratic inequalities	2	0-2	0	4
Inequalities with fractional expressions	2	0-2	0	4

	Text	Exam		
Problem Solving, Word Problems	AVR	RNG	AVR	FOA
General techniques	1	1-1	1	5
Ratio and Proportion	1	0-0	0	5
Percent, Interest	1	0-0	0	5
Geometric formula (Area, etc.)	1	0-1	1	3
Geometry of lines and similar triangles	1	0-0	0	5
Travel (speed, velocity)	1	0-0	0	5
Distance formula in the plane	1	0-2	0	3
Quadratic problem	0	0-0	0	5
Exponential/Log models	0	0-0	0	5
Systems of Equations	1	0-0	0	5
Mixtures	1	0-0	0	5
Exponents	0	0-0	0	5
Roots	1	0-0	0	5

	Text	Exam		
Functions	AVR	RNG	AVR	FOA
Ordered pairs	1	1-1	1	5
Relations, domain, range	1	1-1	1	5
Function notation -- f(x)	3	0-2	1	3
Composition	3	0-2	0	4
Algebra of functions	3	0-0	0	5
Graphing points	1	1-1	1	5
Graphing relations	1	0-0	0	5
Graphing functions	3	0-2	0	3
Vertical line test	3	0-0	0	5
Shift of axes -- f(x) + k, f(x+h)	0	0-0	0	5
Domain/range of functions	3	0-2	0	3
Inverses of relations/functions	2	0-1	0	4
Symmetry about y = x	0	0-0	0	5
One to One functions, horizontal line test	2	0-1	0	3
Formula for an inverse	3	0-2	0	4
Increasing/Decreasing functions	2	0-1	0	4
The difference quotient	2	0-1	0	4

	Text	Exam		
Equations and Inequalities in Two Variables	AVR	RNG	AVR	FOA
Linear equations - graphing, slope	1	1-1	1	5
Lines - Point slope equation	1	0-2	0	4
Lines - Two point equation	1	0-0	0	5
Lines - Point intercept equation	1	0-0	0	5
Parallel and perpendicular lines	1	0-2	0	3
Midpoints of line seqments	1	0-0	0	5
Equations of circles	3	0-2	1	1
Quadratic equations - graphing parabolas	3	0-2	1	1
Quadratic equations - intercepts	3	0-2	1	1
Quadratic equations - vertex, max/min	3	0-2	1	1
Quadratic equations - symmetry	3	0-0	0	5
Roots, Radicals and exponents	1	0-0	0	5

	Text	Exam		
Conic Sections	AVR	RNG	AVR	FOA
Parabola, vertex, focus, directrix (see also Quadratic Equations above.)	3	0-2	1	1
Ellipse -- standard equation/graphing	3	0-2	1	1
Ellipse -- properties	3	0-2	1	1
Hyperbola -- standard equation/graphing	3	0-0	0	5
Hyperbola -- properties	3	0-0	0	5
Translation of axes	3	0-0	0	5
Rotation of axes	0	0-0	0	5
Parametric equations	0	0-0	0	5

	Text	Exam		
Exponential and Log Functions	AVR	RNG	AVR	FOA
Graphs of exponential functions	3	0-0	0	5
Log functions as inverses of exp functions	3	0-0	0	5
Domain and range of exp. and log functions	3	0-0	0	5
Graphs of log functions	3	0-0	0	5
Shift of axes for exp. and log functions	0	0-0	0	5
Conversion: Log ↔ Exp	3	0-2	0	4
Properties of logs	3	0-2	0	4
Change of base	0	0-0	0	5
Natural logs	2	0-0	0	5
Solving equations	3	0-2	0	4

	Text	Exam		
Systems of (mostlyLinear) Eqs/Ineqs, Matrices	AVR	RNG	AVR	FOA
Graphical solution	3	0-0	0	5
Substitution method	3	0-2	1	1
Elimination/Addition method	3	0-2	2	4
Systems with three variables	3	0-2	2	4
Consistent/Inconsistent systems	2	0-0	0	5
Independent/Dependent systems	2	0-0	0	5
Homogeneous systems	2	0-0	0	5
Augmented matrix of a system	3	0-2	2	3
Arithmetic operations on matrices	3	0-2	1	1
System as Ax = b	3	0-2	1	1
Inverses of matrices	3	0-0	0	5
Determinants	3	0-2	2	3
Cramers rule	3	0-2	0	3
Systems of inequalities - Linear programming	3	0-0	0	5

	Text	Exam		
Circles, Triangles, and Angles	AVR	RNG	AVR	FOA
Geometry of a circle: center, radius, diameter, PI, area, circumference, arc, central angle	2	1-1	1	5
Angles -- defined and measured	2	0-2	0	4
Degree and radian measure/conversion	3	0-2	0	4
Central angles, the wrapping function	3	0-2	0	3
Kinds of angles -- straight, right, acute, obtuse	3	0-2	1	1
Right triangles	3	0-2	1	1
Exact coordinates on the unit circle/ 30-60-90 and 45-45-90 right triangles	3	0-2	1	1

	Text	Exam		
Trigonmetric Functions	AVR	RNG	AVR	FOA
Sin, Cos, Tan, Csc, Sec, Cot on right triangles	3	1-1	1	5
Sin x, Cos x, Tan x, Csc x, Sec x, Cot x for x real	3	0-2	2	3
Signs of trig functions in quadrants I to IV	3	0-1	1	3
Exact values of trig functions for special angles	3	0-2	2	4
Solving right triangles	3	0-2	1	2
Solve: sin x = a, cos x = b, tan x = c for one period	0	0-0	0	5
Solve sin x = a, cos x = b, tan x = c for all x	0	0-0	0	0
Word problems	3	0-0	0	0

	Text	Exam		
Trigonmetric Identities	AVR	RNG	AVR	FOA
Reciprocals -- csc = 1/sin, sec = 1/cos, cot = 1/tan	3	0-1	0	3
Quotients -- tan = sin/cos, cot = cos/sin	3	0-1	0	3
Even/Odd -- sin(-x) = -sin x, cos(-x) = cos x, tan(-x) = - tan x	0	0-0	0	5
Pythagoras -- $\sin^2 + \cos^2 = 1$, $\tan^2 + 1 = \sec^2$	2	0-0	0	5
Law of sines and cosines	3	0-2	2	3
Application to vectors	3	0-0	0	5
Vector word problems -- velocity, force	3	0-0	0	5
sin(A ± B), cos(A ± B), tan(A ± B)	0	0-0	0	5
$\cos(\pi/2 - x) = \sin x$, $\sin(\pi/2 - x) = \cos x$, $\tan(\pi/2 - x) = \cot x$	0	0-0	0	5
sin(2A), sin(A/2), cos(2A), cos(A/2)	0	0-0	0	5
tan(2A), tan(A/2)	0	0-0	0	5
sinA cosB, cosA cosB, sinA sinB	0	0-0	0	5
sinA ± sin B, cosA ± cosB	0	0-0	0	0

	Text	Exam		
Graphs of trigonmetric functions	AVR	RNG	AVR	FOA
Periodic functions	0	0-0	0	5
Graph of sin and cos	0	0-0	0	5
Graph of tan and cot	0	0-0	0	5
Graph of csc and sec	0	0-0	0	5
Graph of Asin(Bx + C), Acos(Bx + C)	0	0-0	0	5
Amplitude	0	0-0	0	5
Period	0	0-0	0	5
Phase shift	0	0-0	0	5
sin^{-1}, cos^{-1}	0	0-0	0	5
tan^{-1}	0	0-0	0	5

	Text	Exam		
Polar Coordinates	AVR	RNG	AVR	FOA
Polar coordinates defined	0	0-0	0	5
Relation to rectangular coordinates	0	0-0	0	5
Graphing points	0	0-0	0	5
Graphing curves	0	0-0	0	5
Complex numbers in polar form	0	0-0	0	5

Table 2 Topics for Math 103 AMC
(based on departmental sylabus and one final examination)

Table codes: Depth of coverage based upon test and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered.
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frequency of average.

Real Numbers	Text	Exam
Arith. of signed numbers, additive inverses	1	1
Multiplicative inverses	1	1
Equivalent expressions	1	1
Properties of + and ×	1	1
Integer exponents: notation, laws	1	1
Absolute value (definition and properties)	1	0
Intervals	2	0
Union and intesection of sets	2	0
Arithmetic of complex numbers	0	0

Algebraic Expressions (AEs)	Text	Exam
Polynomials (definition, recognition)	1	1
Addition and subtraction of AEs	1	1
Multiplying polynomials	1	0
Factoring polynomials - common factor	1	1
Factoring polynomials - difference of squares	1	0
Factoring quadratics	1	1
Factoring by quadratic technique	1	0
Factoring special cubic polynomials	1	0
Fractional expressions - simplifying	1	1
Fractional expressions +, -, ×, ÷	1	1
Fractions of fractions -	1	0
Radical notation	1	0
Rational exponents, definition, laws	1	0
Division of polynomials	0	0

	Text	Exam
Polynomials		
Graphs	1	0
Division algorithm	0	0
Roots and factors	1	1
Remainder theorem	0	0
Synthetic division	0	0
Fundamental theorem of algebra	0	0
Polynomials with given roots	0	0

	Text	Exam
Equations and Inequalities in One Unknown		
General principles	1	1
Linear equations	1	1
Quadratic equations - factoring	1	1
Quadratic equations - completing the square	1	1
Quadratic formula - discriminant	1	0
Complex roots of quadratics	0	0
Equations with radicals	1	0
Equations reducible to quadratics	1	0
Equations with fractional expressions	1	0
Equations with absolute value	1	1
Linear inequalities	1	1
Inequalities with absolute value	2	2
Quadratic inequalities	1	0
Inequalities with fractional expressions	0	0

	Text	Exam
Problem Solving, Word Problems		
General techniques	1	0
Ratio and Proportion	1	0
Percent, Interest	1	0
Geometric formula (Area, etc.)	1	0
Geometry of lines and similar triangles	1	1
Travel (speed, velocity)	1	0
Distance formula in the plane	1	1
Quadratic problem	1	0
Exponential/Log models	0	0
Systems of Equations	1	0
Mixtures	0	0
Exponents	0	0
Roots	0	0

Functions	Text	Exam
Ordered pairs	1	1
Relations, domain, range	1	0
Function notation -- f(x)	3	2
Composition	3	2
Algebra of functions	3	2
Graphing points	1	1
Graphing relations	1	0
Graphing functions	3	2
Vertical line test	0	0
Shift of axes -- f(x) + k, f(x+h)	0	0
Domain/range of functions	3	2
Inverses of relations/functions	1	1
Symmetry about y = x	0	0
One to One functions, horizontal line test	1	1
Formula for an inverse	3	2
Increasing/Decreasing functions	0	0
The difference quotient	0	0

Equations and Inequalities in Two Variables	Text	Exam
Linear equations - graphing, slope	1	0
Lines - Point slope equation	1	0
Lines - Two point equation	1	0
Lines - Point intercept equation	1	0
Parallel and perpendicular lines	0	0
Midpoints of line seqments	0	0
Equations of circles	1	0
Quadratic equations - graphing parabolas	1	2
Quadratic equations - intercepts	1	1
Quadratic equations - vertex, max/min	1	1
Quadratic equations - symmetry	1	1
Roots, Radicals and exponents	0	0

Conic Sections	Text	Exam
Parabola, vertex, focus, directrix (see also Quadratic Equations above.)	0	0
Ellipse -- standard equation/graphing	0	0
Ellipse -- properties	0	0
Hyperbola -- standard equation/graphing	0	0
Hyperbola -- properties	0	0
Translation of axes	0	0
Rotation of axes	0	0
Parametric equations	0	0

Exponential and Log Functions	Text	Exam
Graphs of exponential functions	0	0
Log functions as inverses of exp functions	0	0
Domain and range of exp. and log functions	0	0
Graphs of log functions	0	0
Shift of axes for exp. and log functions	0	0
Conversion: Log ↔ Exp	0	0
Properties of logs	0	0
Change of base	0	0
Natural logs	0	0
Solving equations	0	0

Systems of (mostlyLinear) Eqs/Ineqs, Matrices	Text	Exam
Graphical solution	0	0
Substitution method	0	0
Elimination/Addition method	0	0
Systems with three variables	0	0
Consistent/Inconsistent systems	0	0
Independent/Dependent systems	0	0
Homogeneous systems	0	0
Augmented matrix of a system	0	0
Arithmetic operations on matrices	0	0
System as Ax = b	0	0
Inverses of matrices	0	0
Determinants	0	0
Cramèrs rule	0	0
Systems of inequalities - Linear programming	0	0

	Text	Exam
Circles, Triangles, and Angles		
Geometry of a circle: center, radius, diameter, PI, area, circumference, arc, central angle	1	1
Angles -- defined and measured	1	1
Degree and radian measure/conversion	1	1
Central angles, the wrapping function	2	1
Kinds of angles -- straight, right, acute, obtuse	1	1
Right triangles	1	1
Exact coordinates on the unit circle/ 30-60-90 and 45-45-90 right triangles	1	1

	Text	Exam
Trigonmetric Functions		
Sin, Cos, Tan, Csc, Sec, Cot on right triangles	1	1
Sin x, Cos x, Tan x, Csc x, Sec x, Cot x for x real	3	2
Signs of trig functions in quadrants I to IV	3	2
Exact values of trig functions for special angles	3	2
Solving right triangles	1	1
Solve: sin x = a, cos x = b, tan x = c for one period	3	1
Solve sin x = a, cos x = b, tan x = c for all x	3	0
Word problems	3	0

	Text	Exam
Trigonmetric Identities		
Reciprocals -- csc = 1/sin, sec = 1/cos, cot = 1/tan	3	2
Quotients -- tan = sin/cos, cot = cos/sin	3	2
Even/Odd -- sin(-x) = -sin x, cos(-x) = cos x, tan(-x) = - tan x	3	0
Pythagoras -- $\sin^2 + \cos^2 = 1$, $\tan^2 + 1 = \sec^2$	3	2
Law of sines and cosines	1	0
Application to vectors	1	0
Vector word problems -- velocity, force	3	0
sin(A ± B), cos(A ± B), tan(A ± B)	3	2
cos(π/2 - x) = sin x, sin(π/2 - x) = cos x, tan(π/2 - x) = cot x	3	0
sin(2A), sin(A/2), cos(2A), cos(A/2)	3	2
tan(2A), tan(A/2)	3	2
sinA cosB, cosA cosB, sinA sinB	0	0
sinA ± sin B, cosA ± cosB	0	0

	Text	Exam
Graphs of trigonmetric functions		
Periodic functions	3	0
Graph of sin and cos	3	0
Graph of tan and cot	3	0
Graph of csc and sec	0	0
Graph of Asin(Bx + C), Acos(Bx + C)	3	0
Amplitude	0	0
Period	0	0
Phase shift	0	0
$\sin^{-1}$, $\cos^{-1}$	0	0
$\tan^{-1}$	0	0

	Text	Exam
Polar Coordinates		
Polar coordinates defined	3	0
Relation to rectangular coordinates	3	0
Graphing points	3	0
Graphing curves	3	0
Complex numbers in polar form	0	0

Table 3 Math 126 at GSU
(based on six sylabus and five exam responses)

Table codes: Depth of coverage based upon test and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered.
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.
RNG = range, AVR = average, FOA = frenquency of average.

	Text			Exam		
Real Numbers	RNG	AVR	FOA	RNG	AVR	FOA
Arith. of signed numbers, additive inverses	1-1	1	6	1-1	1	5
Multiplicative inverses	1-1	1	6	0-0	0	5
Equivalent expressions	1-1	1	6	1-1	1	5
Properties of + and ×	1-1	1	6	1-1	1	5
Integer exponents: notation, laws	1-1	1	6	0-0	0	5
Absolute value (definition and properties)	1-1	1	6	0-0	0	5
Intervals	1-1	1	6	0-1	0	3
Union and intesection of sets	1-1	1	6	1-1	1	5
Arithmetic of complex numbers	1-1	1	6	0-0	0	5

	Text			Exam		
Algebraic Expressions (AEs)	RNG	AVR	FOA	RNG	AVR	FOA
Polynomials (definition, recognition)	1-1	1	6	1-1	1	5
Addition and subtraction of AEs	1-1	1	6	1-1	1	5
Multiplying polynomials	1-1	1	6	0-1	1	3
Factoring polynomials - common factor	1-1	1	6	0-1	1	3
Factoring polynomials - difference of squares	1-1	1	6	0-1	0	3
Factoring quadratics	1-1	1	6	1-1	1	5
Factoring by quadratic technique	1-1	1	6	0-1	0	4
Factoring special cubic polynomials	1-1	1	6	0-0	0	5
Fractional expressions - simplifying	1-1	1	6	1-1	1	5
Fractional expressions +, -, ×, ÷	1-1	1	6	0-1	1	4
Fractions of fractions -	1-1	1	6	0-1	1	3
Radical notation	1-1	1	6	0-1	0	3
Rational exponents, definition, laws	1-1	1	6	0-0	0	5
Division of polynomials	1-1	1	6	0-0	0	5

	Text			Exam		
Polynomials	RNG	AVR	FOA	RNG	AVR	FOA
Graphs	1-1	1	6	0-0	0	5
Division algorithm	0-0	0	6	0-0	0	5
Roots and factors	1-1	1	6	0-0	0	5
Remainder theorem	0-0	0	6	0-0	0	5
Synthetic division	0-0	0	6	0-0	0	5
Fundamental theorem of algebra	0-0	0	6	0-0	0	5
Polynomials with given roots	0-0	0	6	0-0	0	5

	Text			Exam		
Equations and Inequalities in One Unknown	RNG	AVR	FOA	RNG	AVR	FOA
General principles	1-1	1	6	1-1	1	5
Linear equations	1-1	1	6	1-1	1	5
Quadratic equations - factoring	1-1	1	6	1-1	1	5
Quadratic equations - completing the square	1-1	1	6	0-1	1	3
Quadratic formula - discriminant	0-1	1	3	0-0	0	5
Complex roots of quadratics	0-1	0	4	0-0	0	5
Equations with radicals	0-1	1	4	0-1	0	3
Equations reducible to quadratics	0-1	1	5	0-1	0	3
Equations with fractional expressions	0-1	1	5	0-1	1	4
Equations with absolute value	0-1	1	4	0-0	0	5
Linear inequalities	1-1	1	5	1-1	1	5
Inequalities with absolute value	0-1	0	4	0-0	0	5
Quadratic inequalities	0-1	0	4	0-0	0	5
Inequalities with fractional expressions	0-1	0	4	0-0	0	05

	Text			Exam		
Problem Solving, Word Problems	RNG	AVR	FOA	RNG	AVR	FOA
General techniques	1-1	1	6	0-0	0	5
Ratio and Proportion	1-1	1	6	0-0	0	5
Percent, Interest	0-0	0	6	0-0	0	5
Geometric formula (Area, etc.)	1-1	1	6	0-0	0	5
Geometry of lines and similar triangles	1-1	1	6	0-0	0	5
Travel (speed, velocity)	0-0	0	6	0-0	0	5
Distance formula in the plane	1-1	1	6	0-2	1	2
Quadratic problem	0-0	0	6	0-0	0	5
Exponential/Log models	0-0	0	6	0-0	0	5
Systems of Equations	0-0	0	6	0-0	0	5
Mixtures	0-0	0	6	0-0	0	5
Exponents	0-0	0	6	0-0	0	5
Roots	0-0	0	6	0-0	0	5

	Text			Exam		
Functions	RNG	AVR	FOA	RNG	AVR	FOA
Ordered pairs	0-3	2	2	0-0	0	5
Relations, domain, range	0-3	2	2	0-0	0	5
Function notation -- f(x)	0-3	2	2	0-2	0	3
Composition	0-3	1	3	0-1	0	4
Algebra of functions	0-2	1	3	0-0	0	5
Graphing points	0-3	1	3	1-1	1	5
Graphing relations	0-3	1	3	0-0	0	5
Graphing functions	0-3	3	3	1-2	2	3
Vertical line test	0-3	2	1	0-0	0	5
Shift of axes -- f(x) + k, f(x - h)	0-3	3	4	1-1	1	5
Domain/range of functions	2-3	3	3	0-2	2	3
Inverses of relations/functions	2-3	3	3	0-1	1	4
Symmetry about y = x	1-3	3	3	0-0	0	5
One to One functions, horizontal line test	1-3	3	3	0-0	0	5
Formula for an inverse	2-3	3	3	0-2	2	4
Increasing/Decreasing functions	0-2	1	1	0-0	0	5
The difference quotient	0-3	1	0	0-2	0	4

	Text			Exam		
Equations and Inequalities in Two Variables	RNG	AVR	FOA	RNG	AVR	FOA
Linear equations - graphing, slope	0-3	1	3	0-1	0	3
Lines - Point slope equation	0-2	1	4	0-0	0	5
Lines - Two point equation	0-2	1	4	0-0	0	5
Lines - Point intercept equation	0-2	1	4	0-0	0	5
Parallel and perpendicular lines	0-2	1	3	0-0	0	5
Midpoints of line seqments	0-2	1	3	0-0	0	5
Equations of circles	0-3	1	3	0-2	1	1
Quadratic equations - graphing parabolas	0-3	2	2	0-2	2	3
Quadratic equations - intercepts	0-3	2	2	0-2	0	3
Quadratic equations - vertex, max/min	0-3	2	2	0-2	0	3
Quadratic equations - symmetry	0-3	2	2	0-0	0	5
Roots, Radicals and exponents	0-3	0	3	0-0	0	5

	Text			Exam		
Conic Sections	RNG	AVR	FOA	RNG	AVR	FOA
Parabola, vertex, focus, directrix (see also Quadratic Equations above.)	0-3	3	4	0-2	1	1
Ellipse -- standard equation/graphing	0-3	3	4	0-2	2	3
Ellipse -- properties	0-3	3	4	0-2	2	3
Hyperbola -- standard equation/graphing	0-3	3	4	0-2	0	3
Hyperbola -- properties	0-3	3	4	0-2	0	3
Translation of axes	0-3	0	3	0-1	1	4
Rotation of axes	0-2	0	4	0-0	0	5
Parametric equations	0-2	1	1	0-2	0	4

	Text			Exam		
Exponential and Log Functions	RNG	AVR	FOA	RNG	AVR	FOA
Graphs of exponential functions	0-0	0	6	0-0	0	5
Log functions as inverses of exp functions	0-0	0	6	0-0	0	5
Domain and range of exp. and log functions	0-0	0	6	0-0	0	5
Graphs of log functions	0-0	0	6	0-0	0	5
Shift of axes for exp. and log functions	0-0	0	6	0-0	0	5
Conversion: Log ↔ Exp	0-0	0	6	0-0	0	5
Properties of logs	0-0	0	6	0-0	0	5
Change of base	0-0	0	6	0-0	0	5
Natural logs	0-0	0	6	0-0	0	5
Solving equations	0-0	0	6	0-0	0	5

	Text			Exam		
Systems of (mostlyLinear) Eqs/Ineqs, Matrices	RNG	AVR	FOA	RNG	AVR	FOA
Graphical solution	0-3	1	3	1-1	1	5
Substitution method	0-2	1	3	0-0	0	5
Elimination/Addition method	0-3	1	3	0-0	0	5
Systems with three variables	0-2	0	4	0-0	0	5
Consistent/Inconsistent systems	0-1	0	5	0-0	0	5
Independent/Dependent systems	0-1	0	5	0-0	0	5
Homogeneous systems	0-1	0	4	0-0	0	5
Augmented matrix of a system	0-1	0	5	0-0	0	5
Arithmetic operations on matrices	0-1	0	5	0-0	0	5
System as Ax = b	0-0	0	6	0-0	0	5
Inverses of matrices	0-0	0	6	0-0	0	5
Determinants	0-0	0	6	0-0	0	5
Cramers rule	0-0	0	6	0-0	0	5
Systems of inequalities - Linear programming	3-3	3	6	2-2	2	5

	Text			Exam		
Circles, Triangles, and Angles	RNG	AVR	FOA	RNG	AVR	FOA
Geometry of a circle: center, radius, diameter, PI, area, circumference, arc, central angle	1-3	2	1	0-2	1	1
Angles -- defined and measured	2-3	3	5	1-1	1	5
Degree and radian measure/conversion	3-3	3	6	1-2	1	4
Central angles, the wrapping function	3-3	3	6	0-2	1	2
Kinds of angles -- straight, right, acute, obtuse	1-3	3	4	0-1	1	3
Right triangles	2-3	3	5	1-1	1	5
Exact coordinates on the unit circle/ 30-60-90 and 45-45-90 right triangles	3-3	3	6	1-2	1	4

	Text			Exam		
Trigonmetric Functions	RNG	AVR	FOA	RNG	AVR	FOA
Sin, Cos, Tan, Csc, Sec, Cot on right triangles	3-3	3	6	1-1	1	5
Sin x, Cos x, Tan x, Csc x, Sec x, Cot x, x real	3-3	3	6	1-1	1	5
Signs of trig functions in quadrants I to IV	3-3	3	6	1-2	1	3
Exact values of trig functions for special angles	3-3	3	6	2-2	2	5
Solving right triangles	3-3	3	6	2-2	2	6
Solve: sin x = a, cos x = b, tan x = c for period	3-3	3	6	1-2	2	3
Solve sin x = a, cos x = b, tan x = c for all x	3-3	3	6	1-2	2	3
Word problems	2-3	3	5	0-2	2	4

	Text			Exam		
Trigonmetric Identities	RNG	AVR	FOA	RNG	AVR	FOA
Reciprocals -- csc = 1/sin, sec = 1/cos, cot = 1/tan	3-3	3	6	0-2	1	3
Quotients -- tan = sin/cos, cot = cos/sin	3-3	3	6	1-2	1	4
Even/Odd -- sin(-x) = -sin x, cos(-x) = cos x, tan(-x) = - tan x	3-3	3	6	0-1	0	4
Pythagoras -- $\sin^2 + \cos^2 = 1$, $\tan^2 + 1 = \sec^2$	3-3	3	6	1-1	1	5
Law of sines and cosines	3-3	3	6	0-2	0	4
Application to vectors	0-3	3	4	0-2	0	3
Vector word problems -- velocity, force	0-3	2	2	0-2	0	3
sin(A ± B), cos(A ± B), tan(A ± B)	3-3	3	6	1-2	2	4
$\cos(\pi/2 - x) = \sin x$, $\sin(\pi/2 - x) = \cos x$, $\tan(\pi/2 - x) = \cot x$	3-3	3	6	0-2	0	4
sin(2A), sin(A/2), cos(2A), cos(A/2)	3-3	3	6	1-2	2	4
tan(2A), tan(A/2)	3-3	3	6	0-2	0	3
sinA cosB, cosA cosB, sinA sinB	0-3	3	5	0-0	0	5
sinA ± sin B, cosA ± cosB	0-3	3	4	0-0	0	5

	Text			Exam		
Graphs of trigonmetric functions	RNG	AVR	FOA	RNG	AVR	FOA
Periodic functions	3-3	3	6	1-1	1	5
Graph of sin and cos	3-3	3	6	1-2	2	3
Graph of tan and cot	3-3	3	6	0-2	0	4
Graph of csc and sec	0-3	3	4	0-2	0	4
Graph of Asin(Bx + C), Acos(Bx + C)	3-3	3	6	2-2	2	5
Amplitude	3-3	3	6	1-2	2	3
Period	3-3	3	6	1-2	2	3
Phase shift	3-3	3	6	1-2	2	3
$\sin^{-1}$, $\cos^{-1}$	2-3	3	5	0-2	1	2
$\tan^{-1}$	2-3	3	5	0-1	1	3

	Text			Exam		
Polar Coordinates	RNG	AVR	FOA	RNG	AVR	FOA
Polar coordinates defined	1-3	3	5	0-1	1	4
Relation to rectangular coordinates	1-3	3	5	0-2	1	1
Graphing points	1-3	3	5	0-1	0	3
Graphing curves	1-3	3	5	0-1	0	3
Complex numbers in polar form	0-3	0	3	0-0	0	5

Table 4 Summary for Math 104, 126 at GSU, and
Math 101, 102, 103 at AMC

Table codes: Depth of coverage based upon test and sylabi
0 = No coverage, 1 = assumed, 2 = light coverage, 3 = covered.
Depth of coverage based upon final examinations
0 = Not needed, 1 = Implicit use, 2 = Explicit use.

For each course Average value for Text/Exam from previous tables.

	104	126	101	102	103
Real Numbers					
Arith. of signed numbers, additive inverses	1 1	1 1	1 1	1 1	1 1
Multiplicative inverses	1 1	1 0	1 1	1 0	1 1
Equivalent expressions	1 1	1 1	1 1	1 1	1 1
Properties of + and ×	1 1	1 1	1 1	1 1	1 1
Integer exponents: notation, laws	2 1	1 0	1 1	1 1	1 1
Absolute value (definition and properties)	2 2	1 0	1 1	1 0	1 0
Intervals	2 1	1 0	1 1	1 0	2 0
Union and intesection of sets	3 2	1 1	1 1	1 0	2 0
Arithmetic of complex numbers	3 2	1 0	0 0	3 0	0 0

	104	126	101	102	103
Algebraic Expressions (AEs)					
Polynomials (definition, recognition)	2 1	1 1	2 1	1 1	1 1
Addition and subtraction of AEs	2 1	1 1	2 1	1 1	1 1
Multiplying polynomials	2 1	1 1	2 1	1 1	1 0
Factoring polynomials - common factor	2 2	1 1	2 2	1 1	1 1
Factoring polynomials - difference of squares	2 2	1 0	2 2	1 0	1 0
Factoring quadratics	2 2	1 1	2 2	1 1	1 1
Factoring by quadratic technique	2 2	1 0	2 2	1 0	1 0
Factoring special cubic polynomials	2 2	1 0	2 2	1 0	1 0
Fractional expressions - simplifying	2 2	1 1	2 2	1 0	1 1
Fractional expressions +, -, ×, ÷	2 2	1 1	2 2	1 0	1 1
Fractions of fractions -	2 1	1 1	2 1	1 0	1 0
Radical notation	3 1	1 0	3 1	1 0	1 0
Rational exponents, definition, laws	3 1	1 0	3 1	1 0	1 0
Division of polynomials	2 1	1 0	2 1	0 0	0 0

	104	126	101	102	103
Polynomials					
Graphs	20	10	20	11	10
Division algorithm	30	00	30	00	00
Roots and factors	30	10	30	10	11
Remainder theorem	30	00	30	00	00
Synthetic division	21	00	21	00	00
Fundamental theorem of algebra	30	00	30	00	00
Polynomials with given roots	20	00	20	00	00

	104	126	101	102	103
Equations and Inequalities in One Unknown					
General principles	21	11	21	11	11
Linear equations	32	11	21	11	11
Quadratic equations - factoring	32	11	21	11	11
Quadratic equations - completing the square	32	11	21	11	11
Quadratic formula - discriminant	32	10	21	10	10
Complex roots of quadratics	32	00	21	20	00
Equations with radicals	32	10	21	30	10
Equations reducible to quadratics	32	10	21	00	10
Equations with fractional expressions	32	11	21	10	10
Equations with absolute value	32	10	21	20	11
Linear inequalities	32	11	21	30	11
Inequalities with absolute value	32	00	21	30	22
Quadratic inequalities	32	00	21	20	10
Inequalities with fractional expressions	32	00	21	20	00

	104	126	101	102	103
Problem Solving, Word Problems					
General techniques	31	10	31	11	10
Ratio and Proportion	00	10	32	00	10
Percent, Interest	20	00	00	00	10
Geometric formula (Area, etc.)	21	10	00	11	10
Geometry of lines and similar triangles	11	10	00	10	11
Travel (speed, velocity)	22	00	00	10	10
Distance formula in the plane	32	11	22	10	11
Quadratic problem	32	00	22	00	10
Exponential/Log models	32	00	00	00	00
Systems of Equations	32	00	20	10	10
Mixtures	02	00	00	10	00
Exponents	00	00	32	00	00
Roots	21	00	32	10	00

	104	126	101	102	103
Functions					
Ordered pairs	3 1	2 0	3 2	1 1	1 1
Relations, domain, range	3 2	2 0	1 1	1 1	1 0
Function notation -- f(x)	3 2	2 0	0 0	3 1	3 2
Composition	3 2	1 0	0 0	3 0	3 2
Algebra of functions	3 2	1 0	0 0	3 0	3 2
Graphing points	3 2	1 1	3 2	1 1	1 1
Graphing relations	3 2	1 0	3 2	1 0	1 0
Graphing functions	3 2	3 2	0 0	3 0	3 2
Vertical line test	3 2	2 0	0 0	3 0	0 0
Shift of axes -- f(x) + k, f(x+h)	2 0	3 1	0 0	0 0	0 0
Domain/range of functions	3 2	3 2	0 0	3 0	3 2
Inverses of relations/functions	3 2	3 1	0 0	2 0	1 1
Symmetry about y = x	3 0	3 0	0 0	0 0	0 0
One to One functions, horizontal line test	3 2	3 0	0 0	2 0	1 1
Formula for an inverse	3 2	3 2	0 0	3 0	3 2
Increasing/Decreasing functions	3 2	1 0	0 0	2 0	0 0
The difference quotient	3 1	1 0	0 0	2 0	0 0

	104	126	101	102	103
Equations and Inequalities in Two Variables					
Linear equations - graphing, slope	3 2	1 0	3 2	1 1	1 0
Lines - Point slope equation	3 2	1 0	3 2	1 0	1 0
Lines - Two point equation	3 2	1 0	3 2	1 0	1 0
Lines - Point intercept equation	3 2	1 0	3 2	1 0	1 0
Parallel and perpendicular lines	3 2	1 0	3 2	1 0	0 0
Midpoints of line seqments	3 2	1 0	3 2	1 0	0 0
Equations of circles	3 2	1 1	0 0	3 1	1 0
Quadratic equations - graphing parabolas	3 2	2 2	3 2	3 1	1 2
Quadratic equations - intercepts	3 2	2 0	3 2	3 1	1 1
Quadratic equations - vertex, max/min	3 2	2 0	2 2	3 1	1 1
Quadratic equations - symmetry	3 2	2 0	3 2	3 0	1 1
Roots, Radicals and exponents	3 2	0 0	3 2	1 0	0 0

	104	126	101	102	103
Conic Sections					
Parabola, vertex, focus, directrix (see also Quadratic Equations above.)	3 2	3 1	3 2	3 1	0 0
Ellipse -- standard equation/graphing	0 0	3 2	0 0	3 1	0 0
Ellipse -- properties	0 0	3 2	0 0	3 1	0 0
Hyperbola -- standard equation/graphing	0 0	3 0	0 0	3 0	0 0
Hyperbola -- properties	0 0	3 0	0 0	3 0	0 0
Translation of axes	0 0	0 1	0 0	3 0	0 0
Rotation of axes	0 0	0 0	0 0	0 0	0 0
Parametric equations	0 0	1 0	0 0	0 0	0 0

	104	126	101	102	103
Exponential and Log Functions					
Graphs of exponential functions	3 2	0 0	0 2	3 0	0 0
Log functions as inverses of exp functions	3 2	0 0	0 0	3 0	0 0
Domain and range of exp. and log functions	3 2	0 0	0 0	3 0	0 0
Graphs of log functions	3 2	0 0	0 0	3 0	0 0
Shift of axes for exp. and log functions	3 2	0 0	0 0	0 0	0 0
Conversion: Log ↔ Exp	3 2	0 0	0 0	3 0	0 0
Properties of logs	3 2	0 0	0 1	3 0	0 0
Change of base	3 1	0 0	0 0	0 0	0 0
Natural logs	3 2	0 0	0 0	2 0	0 0
Solving equations	3 2	0 0	0 2	3 0	0 0

	104	126	101	102	103
Systems of (mostlyLinear) Eqs/Ineqs, Matrices					
Graphical solution	3 1	1 1	3 2	3 0	0 0
Substitution method	3 2	1 0	3 2	3 1	0 0
Elimination/Addition method	3 2	1 0	3 2	3 2	0 0
Systems with three variables	3 2	0 0	0 0	3 2	0 0
Consistent/Inconsistent systems	3 1	0 0	2 2	2 0	0 0
Independent/Dependent systems	3 1	0 0	2 2	2 0	0 0
Homogeneous systems	3 1	0 0	0 0	2 0	0 0
Augmented matrix of a system	3 2	0 0	0 0	3 2	0 0
Arithmetic operations on matrices	3 2	0 0	0 0	3 1	0 0
System as Ax = b	3 2	0 0	0 0	3 1	0 0
Inverses of matrices	3 0	0 0	0 0	3 0	0 0
Determinants	0 0	0 0	0 0	3 2	0 0
Cramers rule	0 0	0 0	0 0	3 0	0 0
Systems of inequalities - Linear programming	0 0	3 2	2 2	3 0	0 0

	104	126	101	102	103
Circles, Triangles, and Angles					
Geometry of a circle: center, radius, diameter, PI, area, circumference, arc, central angle	0 0	2 1	0 0	2 1	1 1
Angles -- defined and measured	0 0	3 1	0 0	2 0	1 1
Degree and radian measure/conversion	0 0	3 1	0 0	3 0	1 1
Central angles, the wrapping function	0 0	3 1	0 0	3 0	2 1
Kinds of angles -- straight, right, acute, obtuse	0 0	3 1	0 0	3 1	1 1
Right triangles	0 0	3 1	0 0	3 1	1 1
Exact coordinates on the unit circle/ 30-60-90 and 45-45-90 right triangles	0 0	3 1	0 0	3 1	1 1

	104	126	101	102	103
Trigonmetric Functions					
Sin, Cos, Tan, Csc, Sec, Cot on right triangles	0 0	3 1	0 0	3 1	1 1
Sin x, Cos x, Tan x, Csc x, Sec x, Cot x for x real	0 0	3 1	0 0	3 2	3 2
Signs of trig functions in quadrants I to IV	0 0	3 1	0 0	3 1	3 2
Exact values of trig functions for special angles	0 0	3 2	0 0	3 2	3 2
Solving right triangles	0 0	3 2	0 0	3 1	1 1
Solve: sin x = a, cos x = b, tan x = c for one period	0 0	3 2	0 0	0 0	3 1
Solve sin x = a, cos x = b, tan x = c for all x	0 0	3 2	0 0	0 0	3 0
Word problems	0 0	3 2	0 0	3 0	3 0

	104	126	101	102	103
Trigonmetric Identities					
Reciprocals -- csc = 1/sin, sec = 1/cos, cot = 1/tan	0 0	3 1	0 0	3 0	3 2
Quotients -- tan = sin/cos, cot = cos/sin	0 0	3 1	0 0	3 0	3 2
Even/Odd -- sin(-x) = -sin x, cos(-x) = cos x, tan(-x) = - tan x	0 0	3 0	0 0	0 0	3 0
Pythagoras -- $\sin^2 + \cos^2 = 1$, $\tan^2 + 1 = \sec^2$	0 0	3 1	0 0	2 0	3 2
Law of sines and cosines	0 0	3 0	0 0	3 2	1 0
Application to vectors	0 0	3 0	0 0	3 0	1 0
Vector word problems -- velocity, force	0 0	2 0	0 0	3 0	3 0
sin(A ± B), cos(A ± B), tan(A ± B)	0 0	3 2	0 0	0 0	3 2
cos(π/2 - x) = sin x, sin(π/2 - x) = cos x, tan(π/2 - x) = cot x	0 0	3 0	0 0	0 0	3 0
sin(2A), sin(A/2), cos(2A), cos(A/2)	0 0	3 2	0 0	0 0	3 2
tan(2A), tan(A/2)	0 0	3 0	0 0	0 0	3 2
sinA cosB, cosA cosB, sinA sinB	0 0	3 0	0 0	0 0	0 0
sinA ± sin B, cosA ± cosB	0 0	3 0	0 0	0 0	0 0

	104	126	101	102	103
Graphs of trigonmetric functions					
Periodic functions	0 0	3 1	0 0	0 0	3 0
Graph of sin and cos	0 0	3 2	0 0	0 0	3 0
Graph of tan and cot	0 0	3 0	0 0	0 0	3 0
Graph of csc and sec	0 0	3 0	0 0	0 0	0 0
Graph of Asin(Bx + C), Acos(Bx + C)	0 0	3 2	0 0	0 0	3 0
Amplitude	0 0	3 2	0 0	0 0	0 0
Period	0 0	3 2	0 0	0 0	0 0
Phase shift	0 0	3 2	0 0	0 0	0 0
$\sin^{-1}$, $\cos^{-1}$	0 0	3 1	0 0	0 0	0 0
$\tan^{-1}$	0 0	3 1	0 0	0 0	0 0

	104	126	101	102	103
Polar Coordinates					
Polar coordinates defined	0 0	3 1	0 0	0 0	3 0
Relation to rectangular coordinates	0 0	3 1	0 0	0 0	3 0
Graphing points	0 0	3 0	0 0	0 0	3 0
Graphing curves	0 0	3 0	0 0	0 0	3 0
Complex numbers in polar form	0 0	0 0	0 0	0 0	0 0

Appendix D

Pre- & Post-Student and Faculty Surveys & Classroom Observation Instrument

Name____

Social Security Number_______________________

STUDENT SURVEY (PRE)
Mathematics

Directions: Please fill in your name, student ID number, sex, and birth date on the computer sheet. Record your answers on the computer sheet. Where there are blanks requiring additional information, fill in your answers on this sheet. Several items relate to possible sources of concern that you might have about this course. (INDICATE YOUR RESPONSES ON THE ANSWER SHEET, NOT BELOW.)

I. HOMEWORK

1. On the average, how much time do you expect the homework to take?

   A. Less than ½ hour per day
   B. ½-1 hour per day
   C. 1-2 hours per day
   D. More than 2 hours per day

2. Generally I expect the homework to be:

   A. Easy but takes a lot of time.
   B. Hard and takes a lot of time.
   C. Easy and takes little time.
   D. Hard but takes little time.

3. The amount of time required for homework

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

4. The difficulty of homework

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

5. What do you expect the homework assignments to typically consist of

A. All problems following covered sections of text
B. All problems following covered sections of text and handouts
C. Odd, even or some pattern of problems following covered sections of text
D. Odd, even or some pattern of problems following covered sections of text and handout
E. Only selected problems following covered sections of text and handouts

6. Do you feel that not doing enough homework will relate to your performance in this class ?

A. Not a factor
B. Somewhat a factor
C. A major factor
D. An overwhelming factor
E. I plan to do all my homework

II. TESTS

7. What kinds of questions do you expect on the tests ?

A. Mostly multiple choice with some fill in the blank, definition, true-false and show your work

B. Mostly fill in the blank with some multiple choice, definition, true-false and show your work

C. Mostly definition with some multiple choice, fill in the blank, true-false and show your work

D. Mostly true-false with some multiple choice, definition, fill in the blank and show your work

E. Mostly show your work with some multiple choice, definition, fill in the blank and true-false

8. Not having enough time to finish quizzes and exams.
A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

9. Difficulty of quizzes and exams.

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

10. Do you feel lack of preparation for quizzes/tests will be a factor in your performance in this class ?

    A. Not a factor
    B. Somewhat a factor
    C. Major factor
    D. Overwhelming factor
    E. I will prepare for tests

III. CLASSROOM INTERACTION

11. What do you expect from your instructor as a typical classroom presentation ?

    A. Review and new material presented as a lecture with related examples provided by the instructor

    B. Review and new material presented as a lecture with related examples provided and questions answered as they arise?

    C. Review and new material presented as a lecture, followed by instructor examples and by examples for students to solve

    D. Review and new material presented as a lecture, followed by instructor examples, by examples for students to solve and class discussion

    E. Review and new material presented through instructor posed problems and questions with class discussion guiding the presentation

12. I expect to be able to easily follow the instructor's presentation in my text

    A. Yes B. No

13. Do you expect to take notes in class?

    A. Yes B. No

14. Do you expect to give verbal explanations on how to solve problems in class?

    A. Yes B. No

15. I expect the instructor to show me how to solve problems in class.

    A. Yes B. No

16. Do you expect the instructor to show you how to solve problems outside of class?

    A. Yes   B. No

17. Do you expect to show your work on the board or overhead in class?

    A. Yes   B. No

18. Do you expect to work with other students in class?

    A. Yes   B. No

19. I expect the Mathematical procedures presented to:

    A. Usually cover only the most common textbook method

    B. Usually cover all textbook methods

    C. Usually cover only student methods

    D. Usually cover student and textbook methods

20. I expect the instructor to show how to solve problems using:

    A. Usually only the approach in the text.

    B. Sometimes instructor demonstrated approaches other than the ones in the text

    C. Sometimes student demonstrated approaches other than the ones in the text

    D. Often instructor demonstrated approaches other than the ones in the text

    E. Often student demonstrated approaches other than the ones in the text

21. In class, I expect most often to solve problems.

    A. Individually or along with instructor.
    B. In small groups.
    C. Rarely solve problems in class; examples found in homework or in note.

22. Being called on in class.

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

23. Not having enough direction from the instructor about how or what to study.

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

24. Difficulty following lecture and taking notes.

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

IV. STUDENT PREPARATION FOR THIS COURSE

25. I consider myself prepared for this course.

    A. Yes B. No

26. The last course I had in mathematics was in:

    A. College
    B. 12th grade
    C. 11th grade
    D. 10th grade
    E. Before 10th grade

27. The last mathematics course I took was:

    A. Less than 1 year ago
    B. 1-3 years ago
    C. 4-10 years ago
    D. More than 10 years ago

28. Choose the statement that best describes you:

    A. I know a lot about mathematics.
    B. I know something about mathematics.
    C. I don't know much about mathematics.
    D. I don't know anything about mathematics.

29. Poor instruction in previous math courses.

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

30. Poor background in mathematics.

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

31. Do you feel you left the previous course with necessary skill for this course?

    A. Yes B. No

32. Do you feel you left the previous course with the necessary prerequisites ?

    A. Yes B. No

33. I think my grasp of prerequisite concepts is:

    A. Excellent B. Average C. Poor

34. I think my grasp of computational skill is:

    A. Excellent B. Average C. Poor

35. I think my grasp of problem solving strategies is:

    A. Excellent B. Average C. Poor

36. I think my grasp of course requirements is:

    A. Excellent B. Average C. Poor

37. I think my grasp of required vocabulary is:

    A. Excellent B. Average C. Poor

V. STUDENT EXPECTATIONS AND BELIEFS

38. In mathematics there is usually one right answer.

    A. Yes B. No

39. In mathematics there is usually one way to solve the problem.

    A. Yes B. No

40. Most mathematics problems can be solved in less than 10 minutes.

A. Yes B. No

41. A good mathematics student will "get started" right away on a problem.

A. Yes B. No

42. Choose the most appropriate response.

A. I learn math easily.
B. I can learn new math topics.
C. I can learn new math topics with difficulty.
D. I can't learn math.

43. Heavy course load

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

44. Outside of class responsibilities (work, home, etc.)

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

45. Pace of the course

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

46. The grade I expect is

A. A
B. B
C. C
D. D
E. Failing

47. No question

48. No question

49. I am interested in Math. and motivated to do well in this class.

A. Strongly agree
B. Agree
C. Disagree
D. Strongly disagree

50. My school materials are:

A. Well organized.
B. Somewhat organized.
C. Not very organized.
D. Totally unorganized.

51. I will be able to take good notes in this class.

A. Yes B. No

52. I will read the text in this class:

A. a lot.
B. some.
C. a little.
D. not at all

VI. Demographic Information

53. Ethnic Identity

A. African-American (not of Hispanic origin)
B. Caucasian (not of Hispanic origin)
C. Hispanic
D. Asian or Pacific Islander
E. American Indian or Alaskan native

54. Are you or have you been a Developmental Studies student in other subject areas?

A. Yes B. No

55. Is English your first language?

A. Yes B. No

56. This course was

A. taken voluntarily.
B. required but not needed.
C. required and needed.

VII. Comments

Complete this sentence.

Mathematics is______________________________________

Other concerns: ______________________________________

CONSENT FORM

I understand that I am participating in a research project and that the data obtained from me will be pooled with data collected from others. I also understand my name will not be used when the results of this survey are reported.

____
Signature

Thank you for your assistance.

Name______________________________

Social Security Number______________________

STUDENT SURVEY (POST)

Mathematics

Directions: Please fill in your name, student ID number, sex, and birth date on the computer sheet. Record your answers on the computer sheet. Where there are blanks requiring additional information, fill in your answers on this sheet. Several items relate to possible sources of concern that you might have about this course. (INDICATE YOUR RESPONSES ON THE ANSWER SHEET, NOT BELOW.)

I. HOMEWORK

1. On the average, how much time did you spent on your homework ?

    A. Less than ½ hour per day
    B. 1/2 - 1 hour per day
    C. 1-2 hours per day
    D. More than 2 hours per day

2. Generally the homework was:

    A. Easy but took a lot of time.
    B. Hard and took a lot of time.
    C. Easy and took little time.
    D. Hard but took little time.

3. The amount of time required for homework

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

4. The difficulty of homework

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

5. What did the homework assignment typically consist of

   A. All problems following covered sections of text
   B. All problems following covered sections of text and handouts
   C. Odd, even or some pattern of problems following covered sections of text
   D. Odd, even or some pattern of problems following covered sections of text and handout
   E. Only selected problems following covered sections of text and handouts

6. Did not doing enough homework relate to your performance in this class ?

   A. Not a factor
   B. Somewhat a factor
   C. A major factor
   D. An overwhelming factor
   E. I did all my homework

II. TESTS

7. What kinds of questions were on the tests ?

   A. Mostly multiple choice with some fill in the blank, definition, true-false and show your work

   B. Mostly fill in the blank with some multiple choice, definition, true-false and show your work

   C. Mostly definition with some multiple choice, fill in the blank, true-false and show your work

   D. Mostly true-false with some multiple choice, definition, fill in the blank and show your work

   E. Mostly show your work with some multiple choice, definition, fill in the blank and true-false

8. Not having enough time to finish quizzes and exams

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

9. Difficulty of quizzes and exams
   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

10. Was lack of preparation for quizzes/tests a factor in your performance in this class ?

    A. Not a factor
    B. Somewhat a factor
    C. Major factor
    D. Overwhelming factor
    E. I prepared for tests

III. CLASSROOM INTERACTION

11. What was your instructor's typical classroom presentation ?

    A. Review and new material presented as a lecture with related examples provided by the instructor

    B. Review and new material presented as a lecture with related examples provided and questions answered as they arose.

    C. Review and new material presented as a lecture, followed by instructor examples and by examples for students to solve

    D. Review and new material presented as a lecture, followed by instructor examples, by examples for students to solve and class discussion

    E. Review and new material presented through instructor posed problems and questions with class discussion guiding the presentation

12. I was able to easily follow the instructor's presentation in my text

    A. Yes B. No

13. Did you take notes in class?

    A. Yes B. No

14. Did you give verbal explanations on how to solve problems in class?

    A. Yes B. No

15. Did the instructor show you how to solve problems in class?

    A. Yes B. No

16. Did the instructor show you how to solve problems outside of class?

    A. Yes   B. No

17. Did you show your work on the board or overhead in class?

    A. Yes   B. No

18. Did you work with other students in class?

    A. Yes   B. No

19. The Mathematical procedures presented :

    A. Usually covered only the most common textbook method
    B. Usually covered all textbook methods

    C. Usually covered only student methods

    D. Usually covered student and textbook methods

20. The instructor showed how to solve problems using:

    A. Usually only the approach in the text.

    B. Sometimes instructor demonstrated approaches other than the ones in the text

    C. Sometimes student demonstrated approaches other than the ones in the text

    D. Often instructor demonstrated approaches other than the ones in the text

    E. Often student demonstrated approaches other than the ones in the text

21. In class, I most often solved problems

    A. Individually or along with instructor.
    B. In small groups.
    C. Rarely solved problems in class; examples found in homework or in notes.

22. Being called on in class

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

23. Not having enough direction from the instructor about how or what to study

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

24. Difficulty following lecture and taking notes

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

IV. STUDENT PREPARATION FOR THIS COURSE

25. I was prepared for this course.

A. Yes B. No

26. The last course I had in mathematics was in:

A. College
B. 12th grade
C. 11th grade
D. 10th grade
E. Before 10th grade

27. The last mathematics course I took was:

A. Less than 1 year ago
B. 1-3 years ago
C. 4-10 years ago
D. More than 10 years ago

28. Choose the statement that best describes you:

A. I know a lot about mathematics.
B. I know something about mathematics.
C. I don't know much about mathematics.
D. I don't know anything about mathematics.

29. Poor instruction in previous math courses

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

30. Poor background in mathematics

A. No concern
B. Some concern
C. Great concern
D. Overwhelming concern

31. Did you have the necessary skills for this course?

A. Yes B. No

32. Did you have the necessary prerequisites for this course ?

A. Yes B. No

33. My grasp of prerequisite concepts was:

A. Excellent B. Average C. Poor

34. My grasp of computational skill was:

A. Excellent B. Average C. Poor

35. My grasp of problem solving strategies was:

A. Excellent B. Average C. Poor

36. My grasp of course requirements was:

A. Excellent B. Average C. Poor

37. My grasp of required vocabulary was:

A. Excellent B. Average C. Poor

V. STUDENT EXPECTATIONS AND BELIEFS

38. In mathematics there is usually one right answer.

A. Yes B. No

39. In mathematics there is usually one way to solve the problem.

A. Yes B. No

40. Most mathematics problems can be solved in less than 10 minutes.

    A. Yes B. No

41. A good mathematics student will "get started" right away on a problem.

    A. Yes B. No

42. Choose the most appropriate response.

    A. I learn math easily.
    B. I can learn new math topics.
    C. I can learn new math topics with difficulty.
    D. I can't learn math.

43. Heavy course load

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

44. Outside of class responsibilities (work, home, etc.)

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

45. Pace of the course

    A. No concern
    B. Some concern
    C. Great concern
    D. Overwhelming concern

46. The grade I expect is

    A. A
    B. B
    C. C
    D. D
    E. Failing

47. No question

48. No question

49. I was interested in this class and motivated to do well in it.

    A. Strongly agree
    B. Agree
    C. Disagree
    D. Strongly disagree

50. My school materials in this class were:

    A. Well organized.
    B. Somewhat organized.
    C. Not very organized.
    D. Totally unorganized.

51. I was able to take good notes in this class.

    A. Yes B. No

52. I did read the text in this class.

    A. a lot.
    B. some.
    C. a little.
    D. not at all.

VI. Demographic Information

53. Ethnic Identity

    A. African-American (not of Hispanic origin)
    B. Caucasian (not of Hispanic origin)
    C. Hispanic
    D. Asian or Pacific Islander
    E. American Indian or Alaskan native

54. Are you or have you been a Developmental Studies student in other subject areas?

    A. Yes B. No

55. Is English your first language?

    A. Yes B. No

56. This course was

    A. taken voluntarily.
    B. required but not needed.
    C. required and needed.

VII. Comments

Complete this sentence.

Mathematics is ____

Other concerns (Describe): ____________________________________

CONSENT FORM

I understand that I am participating in a research project and that the data obtained from me will be pooled with data collected from others. I also understand my name will not be used when the results of this survey are reported.

____
Signature

Thank you for your assistance.

FACULTY SURVEY (PRE)
Mathematics

HOMEWORK

1. Homework should typically take:

   A. Less than ½ hour per day
   B. 1/2 - 1 hours per day
   C. 1-2 hours per day
   D. Over 2 hours per day

2. I expect students to find the homework to be

   A. Easy but requiring a lot of time
   B. Hard and requiring a lot of time
   C. Easy and requiring little time
   D. Hard and requiring little time

3. Student concern about time required for homework will be

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

4. Student concern about difficult of homework will be

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

5. Homework assignments typically consist of:

   A. All problems following pertinent sections of text.
   B. All problems following pertinent sections of text along with handouts.
   C. Odd, even, or other pattern of problems following pertinent sections of text.
   D. Odd, even, or other pattern of problems following pertinent sections of text along with handouts.
   E. Only selected problems following pertinent sections of text.
   F. Only selected problems following pertinent sections of text with additional handouts
   G. Other
   (Explain)____

   ____

6. Do you feel not doing enough homework relates to poor performance of students in this course?

   A. Not a factor
   B. Somewhat a factor
   C. A major factor
   D. An overwhelming factor

TESTS

7. Give a percentage breakdown of general structure of tests for this course.

   A. Multiple choice ______%
   B. Fill in blank ______%
   C. Definition ______%
   D. True/False ______%
   E. Show your work (graded) ______%
   F. Other (explain) ________________________________ ______%

8. Will students have enough time to complete tests/quizzes ?

   A. Yes
   B. No

9. I expect student concern about difficult of quizzes/exams to be:

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern

10. Do you feel lack of preparation for tests relates to poor performance of students in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

CLASSROOM INTERACTION

11. Which description most describes your planned typical classroom presentation? (Select one)

    A. Review and new material presented as a lecture with related examples provided by the instructor?

    B. Review and new material presented as a lecture with related examples provided and questions answered as they arise?

    C. Review and new material presented as a lecture, followed by instructor examples and by examples for students to solve?

    D. Review and new material presented as a lecture, followed by instructor examples, examples for students to solve and class discussion?

    E. Review and new material presented through instructor posed problems and questions with class discussion guiding the presentation?

12. Presentations usually closely follow the information as outlined in the text.

    A. Yes B. No

13. I expect students to take notes

    A. Yes B. No

14. I expect students to give verbal explanations on problem solving

    A. Yes B. No

15. I expect to show students how to solve problems in class

    A. Yes B. No

16. I expect to show students how to solve problems outside of class

    A. Yes B. No

17. I expect students to show their work on overhead or board

    A. Yes B. No

18. I expect students to work with other students in class

    A. Yes B. No

19. When presenting a mathematical procedure:

    A. Usually only the most common textbook method is covered.
    B. Usually all textbook methods are covered.
    C. Usually only student methods are covered.
    D. Usually student and textbook methods are covered.

20. When solving problems:

    A. Usually only the textbook approach to solving the problem is covered.
    B. Usually solution approaches other than those in the text are sometimes provided by the instructor.
    C. Usually solution approaches other than those in the text are sometimes provided by students.
    D. Usually solution approaches other than those in the text are often provided by the instructor.
    E. Usually solution approaches other than those in the text are often presented by students.

21. In class students most often solve problems

    A. Individually or along with instructor.
    B. In small groups.
    C. Rarely solve problems in class; examples found in homework or in notes.

22. I expect to call on students in class

    A. Yes B. No

23. when giving directions about studying, I expect to be:

    A. Specific as to how and what
    B. Specific as to how but not what
    C. Specific as to what but not how
    D. I will give very little direction about studying

24. I expect students to have difficulty

    A. Following lecture and taking notes
    B. Following lecture but not taking notes
    C. Taking notes but not following lecture
    D. No difficulty apparent

34. Students' grasp of prerequisite computational skills is:

A. Excellent B. Average C. Poor

35. Students' grasp of problem-solving strategies is:

A. Excellent B. Average C. Poor

36. Students' grasp of course requirements is:

A. Excellent B. Average C. Poor

37. Students' grasp of required vocabulary is:

A. Excellent B. Average C. Poor

STUDENT EXPECTATIONS

38. I expect the students to feel that in Mathematics there is usually one right answer.

A. I strongly agree with this statement
B. I agree with this statement
C. I disagree with this statement
D. I strongly disagree with this statement
E. I have no opinion on this matter

39. I expect the students to feel that there is usually one way to solve a problem.

A. I strongly agree with this statement
B. I agree with this statement
C. I disagree with this statement
D. I strongly disagree with this statement
E. I have no opinion on this matter

40. I expect the students to feel that most Mathematics problems can be solve in less than 10 minutes.

A. I strongly agree with this statement
B. I agree with this statement
C. I disagree with this statement
D. I strongly disagree with this statement
E. I have no opinion on this matter

STUDENT PREPARATION

25. What percentage of your students are prepared for this math course?

    A. 75 - 100%
    B. 50 - 74%
    C. Less than 50%

26. No question

27. No question

28. No question

29. I expect poor instruction in previous course to affect performance in this course

    A. No factor
    B. Somewhat a factor
    C. Major factor
    D. Overwhelming factor

30. I expect poor background in Mathematics to affect performance in this class

    A. No factor
    B. Some factor
    C. Major factor
    D. Overwhelming factor

31. Do you feel that students leaving the previous course without necessary skills is a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

32. Do you feel lack of prerequisites is a factor in poor performance of students in the course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

33. Students' grasp of prerequisite concepts is:

    A. Excellent B. Average C. Poor

41. I expect the students to feel that a good Mathematics student will get started right away on a problem.

    A. I strongly agree with this statement
    B. I agree with this statement
    C. I disagree with this statement
    D. I strongly disagree with this statement
    E. I have no opinion on this matter

FACULTY PERCEPTIONS

42. Do you feel lack of confidence in math is a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

43. Do you feel having a heavy course load is a factor in poor performance in the course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

44. Do you feel that students' nonacademic responsibilities is a factor in poor performance in the course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

45. Do you feel that the pace of course is a factor in poor performance?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

46. Student expectations seem to be underestimated for:

    A. Grade
    B. Work
    C. Content coverage
    D. None of the above
    E. All of A-C

47. Student expectations seem to be overestimated for:

    A. Grade
    B. Work
    C. Content coverage
    D. None of the above
    E. All of A-C

48. Student expectations seem to be reasonable for:

    A. Grade
    B. Work
    C. Content coverage
    D. None of the above
    E. All of A-C

49. Do you feel that lack of motivation and interest is a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

50. Do you feel the students' inexperience in organizing materials is a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

51. Do you feel that students' ineptness at taking notes is a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

52. Do you think that not reading the text enough is a factor in poor performance in this course?

A. Not a factor
B. Somewhat a factor
C. A major factor
D. An overwhelming factor

How do you use homework, class discussion, and tests to help structure your class?

General Comments

FACULTY SURVEY (Post)
Mathematics

HOMEWORK

1. Homework should typically have taken:

   A. Less than ½ hour per day
   B. ½-1 hours per day
   C. 1-2 hours per day
   D. Over 2 hours per day

2. I feel students found the homework to be

   A. Easy but required a lot of time
   B. Hard and required a lot of time
   C. Easy and required little time
   D. Hard and required little time

3. Student concern about time required for homework was

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern
   E. Students did not comment

4. Student concern about difficulty of homework was

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern
   E. Students did not comment

5. Homework assignments typically consisted of:

   A. All problems following pertinent sections of text.
   B. All problems following pertinent sections of text along with handouts.
   C. Odd, even, or other pattern of problems following pertinent sections of text.
   D. Odd, even, or other pattern of problems following pertinent sections of text along with handouts.
   E. Only selected problems following pertinent sections of text.
   F. Only selected problems following pertinent sections of text with additional handouts
   G. Other
   (Explain)____

   ____

6. Do you feel not doing enough homework related to poor performance of students in this course?

   A. Not a factor
   B. Somewhat a factor
   C. A major factor
   D. An overwhelming factor

TESTS

7. Give a percentage breakdown of general structure of tests for this course.

   A. Multiple choice ______%
   B. Fill in blank ______%
   C. Definition ______%
   D. True/False ______%
   E. Show your work (graded) ______%
   F. Other (explain) ________________________________ ______%

8. Did students have enough time to complete tests/quizzes ?

   A. Yes B. No

9. Student concern about difficult of quizzes/exams was:

   A. No concern
   B. Some concern
   C. Great concern
   D. Overwhelming concern
   E. Students did not comment

10. Do you feel lack of preparation for tests related to poor performance of students in this course?

   A. Not a factor
   B. Somewhat a factor
   C. A major factor
   D. An overwhelming factor

CLASSROOM INTERACTION

11. Which description most describes your typical classroom presentation? (Select one)

    A. Review and new material presented as a lecture with related examples provided by the instructor.

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    C. Review and new material presented as a lecture, followed by instructor examples and by examples for students to solve.

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    E. Review and new material presented through instructor posed problems and questions with class discussion guiding the presentation.

12. Presentations usually closely followed the information as outlined in the text.

    A. Yes B. No

13. Students took notes in class.

    A. Yes B. No

14. Students gave verbal explanations on problem solving in class.

    A. Yes B. No

15. I showed students how to solve problems in class.

    A. Yes B. No

16. I showed students how to solve problems outside of class.

    A. Yes B. No

17. Students showed their work on overhead or board in class.

    A. Yes B. No

18. Students worked together in class.

    A. Yes B. No

19. When presenting a mathematical procedure:

    A. Usually only the most common textbook method was covered.
    B. Usually all textbook methods were covered.
    C. Usually only student methods were covered.
    D. Usually student and textbook methods were covered.

20. When solving problems:

    A. Usually only the textbook approach to solving the problem was covered.
    B. Usually solution approaches other than those in the text were sometimes provided by the instructor.
    C. Usually solution approaches other than those in the text were sometimes provided by students.
    D. Usually solution approaches other than those in the text were often provided by the instructor.
    E. Usually solution approaches other than those in the text were often presented by students.

21. In class students most often solved problems

    A. Individually or along with instructor.
    B. In small groups.
    C. Rarely solved problems in class; examples found in homework or in notes.

22. I called on students in class.

    A. Yes   B. No

23. When giving directions about studying, I was:

    A. Specific as to how and what
    B. Specific as to how but not what
    C. Specific as to what but not how
    D. I gave very little direction about studying

24. Students seemed to have difficulty:

    A. Following lecture and taking notes
    B. Following lecture but not taking notes
    C. Taking notes but not following lecture
    D. No difficulty apparent

STUDENT PREPARATION

25. What percentage of your students were prepared for this math course?

    A. 75 - 100%
    B. 50 - 74%
    C. Less than 50%

26. No question

27. No question

28. No question

29. Poor instruction in previous course affected performance in this course.

    A. No factor
    B. Somewhat a factor
    C. Major factor
    D. Overwhelming factor

30. Poor background in Mathematics affected performance in this class.

    A. No factor
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    B. Somewhat a factor
    C. A major factor
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34. Students' grasp of prerequisite computational skills was:

A. Excellent B. Average C. Poor

35. Students' grasp of problem-solving strategies was:

A. Excellent B. Average C. Poor

36. Students' grasp of course requirements was:

A. Excellent B. Average C. Poor

37. Students' grasp of required vocabulary was:

A. Excellent B. Average C. Poor

STUDENT EXPECTATIONS

38. The students felt that in Mathematics there is usually one right answer.

A. I strongly agree with this statement
B. I agree with this statement
C. I disagree with this statement
D. I strongly disagree with this statement
E. I have no opinion on this matter

39. The students felt that there is usually one way to solve a problem.

A. I strongly agree with this statement
B. I agree with this statement
C. I disagree with this statement
D. I strongly disagree with this statement
E. I have no opinion on this matter

40. The students felt that most Mathematics problems can be solve in less than 10 minutes.

A. I strongly agree with this statement
B. I agree with this statement
C. I disagree with this statement
D. I strongly disagree with this statement
E. I have no opinion on this matter

41. The students felt that a good Mathematics student will " get started" right away on a problem.

    A. I strongly agree with this statement
    B. I agree with this statement
    C. I disagree with this statement
    D. I strongly disagree with this statement
    E. I have no opinion on this matter

FACULTY PERCEPTIONS

42. Do you feel lack of confidence in math was a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

43. Do you feel having a heavy course load was a factor in poor performance in the course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

44. Do you feel that students' nonacademic responsibilities was a factor in poor performance in the course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

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    A. Grade
    B. Work
    C. Content coverage
    D. None of the above
    E. All of A-C

47. Student expectations seemed to be overestimated for:

    A. Grade
    B. Work
    C. Content coverage
    D. None of the above
    E. All of A-C

48. Student expectations seemed to be reasonable for:

    A. Grade
    B. Work
    C. Content coverage
    D. None of the above
    E. All of A-C

49. Do you feel that lack of motivation and interest was a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

50. Do you feel the students' inexperience in organizing materials was a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

51. Do you feel that students' ineptness at taking notes was a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

52. Do you think that not reading the text enough was a factor in poor performance in this course?

    A. Not a factor
    B. Somewhat a factor
    C. A major factor
    D. An overwhelming factor

How did you use homework, class discussion, and tests to help structure your class?

General Comments

OBSERVATION INSTRUMENT

Date:
Class:
Length of class:
Location:
Instructor:
Observation this week:
Topic of day:

I. HOMEWORK

A. Previous Homework

i. Number of problems assigned __________

ii. Number of problems assigned discussed in class ____________

1. Instructor discusses previous homework YES NO
2. Problems are selected by instructor YES NO
3. Problems are selected by students YES NO
4. Student(s) requests explanation of homework (show me how to do it) YES NO
5. Instructor talks through the problem(s) YES NO
6. Student talks through the problem(s) YES NO
7. Instructor provides answers only YES NO
8. Homework is from text YES NO
    i. number of computation problems ____________
    ii. number of word problems ____________
9. Home work is from supplementary books and materials YES NO
10. Homework is from instructor-made problems YES NO

Describe homework assignment

B. Homework for next class

i. Number of problems assigned ________

ii. Number of problems assigned discussed in class ________

11. Instructor discusses homework for next class YES NO

12. Problems are selected by instructor YES NO

13. Problems are selected by students YES NO

14. Homework is from text YES NO

15. Homework is from supplementary books and materials YES NO

16. Homework is from instructor-made problems YES NO

Describe homework assignment

____

____

____

____

II. TESTS (no observation items)

17. Test given

III. CLASSROOM INTERACTION

A. Instructor to students:

18. Instructor probes for specific causes of difficulties related to homework or previous material YES NO

19. Instructor requests or demonstrates a variety of strategies to solve a problem YES NO

20. Instructor asks to predict results or outcome YES NO

21. Instructor asks to recall previous knowledge YES NO

22. Instructor asks to synthesize ideas YES NO

B. Students to Instructor

23. Student(s) asks clarification about overall objectives of the current lesson YES NO

24. Student(s) asks clarification about objectives of previous lesson YES NO

25. Student(s) asks for elaboration of a specific concept/idea YES NO

26. Student(s) asks for elaboration of a specific procedure YES NO

27. Student(s) asks other question(s) YES NO

Give examples ____

____

____

IV. CLASS ATMOSPHERE

28. Active student participation (primarily discussion and questions) YES NO

29. Passive student participation (primarily note taking only) YES NO

30. Participation is encouraged YES NO

V. STUDENT PREPARATION (no items)

VI. STUDENT EXPECTATIONS (no items)

VII. PREPARATION FOR NEXT COURSE (no items)

VIII. OTHER (no items)

IX. MODES OF COMMUNICATION EMPLOYED BY THE INSTRUCTOR:

A. Hand-out ______%

B. Use of projector ______%

i. Previously prepared ______%

ii. Developed in class ______%

C. Use of board ______%

D. Other ______%

Desribe____

____

____

X. CLASS CONTENT

A. Topic(s) of the day (from syllabus)
Sections:

B. Number of sections/pages actually covered
(pages ______ through ______)

31. Instructor presents topic(s) of the day YES NO

XI. DEMOGRAPHIC INFORMATION

A. Number of students ______

B. Number of males/females / ______/______

C. Approximate ages: 18-25 ______ over 25 ______

PERCENTAGES

Time spent discussing assigned homework __________%

Time spent discussing new topic __________%

Time spent discussing new homework __________%

Time spent answering students' questions ___________%

Instructor________________________

Course________________________

Date________________________

Observer________________________

Classroom Observation Summary Items

Please respond to the following items in writing and submit this summsry with the completed observation instrument.

1. Describe the topic of the day and how it was addressed (lecture, group work, student board-work, worksheets, other, some combination) and describe the atmosphere of the class.

2. Describe the types of questions asked by students and instructor and the types of responses given by students and instructor.

3. Describe the balance of time spent between covering homework and presenting new material.

4. Describe any events that caught your attention for this class (dialogue, discussion, activities, other).

5. Please share any other comments or impressions you would like about this observation.

Appendix E

College Algebra Inventory

College Algebra Inventory

by

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College Algebra Inventory

Introduction.

The College Algebra Inventory is an annotated list of operational skills or topics found in most college algebra courses. Each skill has a short description and in most cases is illustrated with an example. Additional samples are provided for some topics in order to explore variations in techniques or levels of difficulty. The skills and topics are arranged in nine broad areas:

Real Numbers
Algebraic Expressions
Equations and Inequalities in One Variable
Problem Solving, Word Problems
Functions
Equations and Inequalities in Two Variables
Exponential and Logarithmic Functions
Systems of Equations/Inequalities and Matrices
Polynomials.

This arrangement is solely for the purpose of classification, and is not suggested as an order of presentation.

The inventory was originally developed as one of several tools for comparing college algebra courses at different schools,and for assessing student's qualifications for further coursework requiring college algebra as a prerequisite. There are several other possible uses for this inventory, including the design and validation of course outlines, textbooks, examinations, and methods of instruction. It can be used by students for self evalution, and by course administrators as a means of achieving uniformity among large numbers of sections. A summary list of the skills and topics is provided at the end of the inventory as an additional tool which may be helpful for some of these purposes.

Real Numbers

Real Numbers Kinds

Description. Recognize the number types: natural, integer, rational, irrational, positive, and negative.

Examples. The natural numbers are $1, 2, 3, \ldots$
The integers are $0, \pm 1, \pm 2, \ldots$
Ratios of integers and repeating decimals are rational numbers.
π, $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ are irrational.

Real Numbers Arithmetic of signed numbers

Description. Use basic rules for the addition, subtraction, multiplication, and division of signed numbers.

Examples. $5 + (-2) = 3$, $(-3) - (-7) = 4$
$(-2) \times 7 = -14$ $(-12) / (-4) = 3$

Real Numbers Properties of + and ×

Description. Use the following properties to manipulate algebraic expressions:

i) $a + (b + c) = (a + b) + c$ ii) $a(bc) = (ab)c$
iii) $a + b = b + a$ iv) $ab = ba$
v) $a + 0 = 0 + a = a$ vi) $1a = a1 = a$
vii) Each real number a has an additive inverse $-a$
viii) Each nonzero real number has a multiplicative inverse a^{-1}
ix) $a(b + c) = ab + ac$ x) $(-1)a = -a$
xi) $-(-a) = a$ xii) $0a = 0$.

Examples. i) $3 + (4 + x) = (3 + 4) + x = 7 + x$ ii) $2(3x) = (2 \cdot 3)x = 6x$
iii) $3 + x + 4 = 3 + 4 + x = 7 + x$ iv) $2x3 = 2 \cdot 3x = 6x$
vii) $(x + 3) + (-3) = x$ viii) $2^{-1}(2x) = x$
ix) $2x + 5x = (2 + 5)x = 7x$ x) $-(x + 3) = -x - 3$

Samples. Simplify: $(3x + 4) - (x - 5)$, $(-2)(-x + 4)$, $[(3x + 4) - 4](3^{-1})$

Real Numbers Order

Description. Interpret the relations $a < b$ and $a \le b$ on the graph of the real number line, and use the properties i) $a < b$ and $b < c \Rightarrow a < c$, ii) $a \le b$ and $b \le a \Rightarrow a = b$

Example. From $a < x - y \le 0$, and $0 \le x - y < b$ we can conclude that $x = y$, and $a < b$. On a number line, a would appear to the left of 0, and b would appear to the right of 0.

Real Numbers Scientific notation

Description. Convert between decimal and scientific notation.

Examples. $12{,}300 = 1.23\times10^{4}$, $1.23\times10^{-2} = .0123$

Real Numbers Equivalent expressions

Description. Recognize when two expressions are equivalent, that is, they yield the same value for any meaningful replacement(s).

Examples. $\frac{2+x-3+4x}{2x+2} = \frac{5x-1}{2(x+1)}$ $\frac{x(x-y)}{x(x+y)} = \frac{x-y}{x+y}$

Real Numbers Order of operations

Description. Use "order of operation" rules for simplifying expressions.

Example. $1 + 3 \times 2 = 1 + 6 = 7$

Real Numbers Integer exponents

Description. (i) Interpret notation for exponents, and use the following properties defining notation for exponents: $a^0 = 1$ $(a \ne 0)$, $a^1 = a$, $a \cdot a^n = a^{n+1}$, $a^{-1} = 1/a$.
(ii) Use the following laws of exponents to simplify expressions containing integer exponents: $a^m \cdot a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = (1/a)^n$, $a^m/a^n = a^{m-n}$, $(ab)^n = a^n b^n$, $(a/b)^n = a^n/b^n$.

Examples. (i) $x \cdot x \cdot x \cdot x = x^4$, $x \cdot x^5 = x^6$, $y^{3-2} = y$.

(ii) $(2ax^2)^3 = 8a^3x^6$, $\frac{x^5y^2}{x^3y^{-1}} = x^2y^3$,

Algebraic Expressions

Algebraic Expressions — Polynomials

Description. Use the definition of polynomial to recognize when an algebraic expression is a polynomial.

Examples. The expression $-5x^4 + x^3 + 6.8x^2 + \sqrt{7}\,x - 3$ is a polynomial in one variable, while, $a^5 - 2a^3 b^4 + 4a^2b^3 + 9b^2 - 3$ is a polynomial in two variables.
$1/(x-2)$, 2^x, and $\sqrt{x-4}$ are not polynomials.

Samples. Which of the following algebraic expressions are polynomials:
$7x^2 + 4x + 1/x$, $x^3 - 5x^2 + 1/3$, $8x^9 + 2x^2 + \sqrt{x}$, $2p^2t^4 + 8pt + 5p^2$?

Algebraic Expressions — Addition and Subtraction

Description. Use the commutative, associative, and distributive laws to "combine" similar terms of two or more algebraic expressions. When subtracting, add the opposite of the expression being subtracted.

Examples. (i) Add $6x^2y - 3xy^2 + 5xy - 3$ and $-4x^2y + x^2y^2 + 2xy + 8$
(ii) Subtract $-3x^4 + 4x^2 + 3$ from $4x^3 - 7x^2 + 8$.

Solution: (i) $(6x^2y - 4x^2y) + x^2y^2 - 3xy^2 + (5xy + 2xy) + (-3 + 8)$
$= 2x^2y + x^2y^2 - 3xy^2 + 7xy + 5.$
(ii) $(4x^3 - 7x^2 + 8) - (-3x^4 + 4x^2 + 3)$
$= 3x^4 + 4x^3 + (-7x^2 - 4x^2) + (8 - 3) = 3x^4 + 4x^3 + -11x^2 + 5.$

Samples. Add $2x + 4y + 3$, $x + 2y - 4$, and $3x - 3y + 2$.
Add $7x^2 + x\sqrt{y} + 3xy$ and $7y^2 + 2x\sqrt{y} - 5xy$.
Subtract $9x^2 - xy - 3y^2$ from $5x^2 + 4xy - y^2$.

Real Numbers Absolute value

Description. Use the definition and properties of absolute value to simplify expressions.

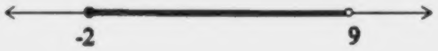
Examples. Given that $a > 0$, $b < 0$, and that x can be any real number:
$|-7ax^2| = 7ax^2$, $|4x/b| = -4|x|/b$.

Real Numbers Intervals

Description. Describe intervals by graphic, set-builder, and interval notation, Convert between descriptions.

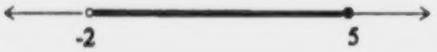
Examples. Describe the set of all real numbers less than 9, and greater than or equal to -2 in three ways.

Solutions. Graphic:



Set-builder: $\{ x : -2 \leq x < 9 \}$
Interval: $[-2, 9)$.

Samples. Describe each of the following sets by two alternative methods:
$\{ x : -3 < x < 2 \}$, $(-\infty, 5)$, $[0, 3]$, $[1, \infty)$,



Real Numbers Union and intersection of sets

Description. Find the union and intersection of finite sets and intervals of real numbers.

Examples.
$\{a, c, e, g\} \cup \{ a, b, f, g\} = \{a, b, c, e, f, g\}$
$\{a, c, e, g\} \cap \{ a, b, f, g\} = \{a, g\}$
$[1, 4) \cup (2, 7] = [1, 7]$
$\{ x : 1 < x < 6 \} \cap \{ x : 3 \leq x < 8 \} = \{ x : 3 \leq x < 6 \}$
$\{ x : x < 3\} \cap (\{-3, -2, -1\} \cup \{ x : 0 < x \} = \{-3, -2, -1\} \cup \{x : 0 < x < 3 \}$

Real Numbers Arithmetic of complex numbers

Description. Use the a + bi notation for complex numbers, where a and b are real, and $i^2 = -1$, to perform arithmetic operations and simplify expressions.

Examples.
$\sqrt{(-9)} - \sqrt{(-25)} = 3i - 5i = -2i$,
$(1 + 2i)(3 + 4i) = -5 + 10i$,

$$\frac{3+4i}{1-2i} = \frac{(3+4i)(1+2i)}{(1-2i)(1+2i)} = \frac{-5+10i}{5} = -1 + 2i$$

Algebraic Expressions — Multiplying polynomials

Description i) Use the distributive laws to multiply polynomials. ii) Use "FOIL" to multiply two binomials. iii) Use the identity $(A + B)^2 = A^2 + 2AB + B^2$ to square a binomial. iv) Use the identity $(A + B)(A - B) = A^2 - B^2$ to find the product of a sum and difference of the same two expressions. v) Use the identify $(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$ to cube a binomial.

Examples. Multiply: i) $(3ax^2 - 4ax + 5)(2x + 3a)$

ii) $(3x + 4y)(x - 2y)$

iii) $(2x^2 - 3y)^2$

iv) $(3x + 2\sqrt{a})(3x - 2\sqrt{a})$

v) $(2x + y^2)^3$

Solution: i) $(3ax^2 - 4ax + 5)(2x + 3a) = (3ax^2 - 4ax + 5)2x + (3ax^2 - 4ax + 5)3a$
$= 6ax^3 - 8ax^2 + 10x + 9a^2x^2 - 12a^2x + 15a.$

ii) $(3x + 4y)(x - 2y) = 3x^2 - 6xy + 4xy - 8y^2 = 3x^2 - 2xy - 8y^2.$

iii) $(2x^2 - 3y)^2 = 4x^4 - 12x^2y + 9y^2.$

iv) $(3x + 2\sqrt{a})(3x - 2\sqrt{a}) = 9x^2 - 4a.$

v) $(2x + y^2)^3 = 8x^3 + 12x^2y^2 + 6xy^4 + y^6.$

Algebraic Expressions — Factoring polynomials - common factor

Description. Use the distributive law to "factor out" any expression common to each term of a polynomial.

Example. Factor $15a^2x^3 + 10ax^2y$

Solution: $15a^2x^3 + 10ax^2y = 5ax^2(3ax + 2y)$

Samples: Factor $6ax^2y^2 - 3axy^3 + 9a^2x^2$, $(bx + x) - (3b + 3)$

Algebraic Expressions — Factoring polynomials - difference of squares

Description. Use the identity $(A + B)(A - B) = A^2 - B^2$ to factor differences of squares.

Example. Factor $4x^2 - a^4y^2$.

Solution: $4x^2 - a^4y^2 = (2x + a^2y)(2x - a^2y).$

Samples. Factor $x^2 - 4$, $x^2 - 3$, $4x^2 - 9(y + a)^2$.

Algebraic Expressions — Factoring quadratics

Description. Use "trial and error" to factor quadratic trionomials and check by using FOIL.

Example. Factor $5x^2 - 18x - 8$.

Solution: $5x^2 - 18x - 8 = (5x + 2)(x - 4)$.

Samples. Factor $x^2 + 9x + 14$, $x^2 - 9x + 14$, $x^2 + 5x - 14$, $3x^2 + 8x - 3$.

Algebraic Expressions — Factoring by quadratic technique

Description. Recognize when an algebraic expression may be reduced to a quadratic in one variable by some substitution, and obtain the corresponding factorization of the original expression.

Example. Factor $x^4 + 5x^2 + 6$

Solution: With $u = x^2$, $x^4 + 5x^2 + 6 = u^2 + 5u + 6 = (u + 2)(u + 3) = (x^2 + 2)(x^2 + 3)$

Samples. Factor $x^{2/3} - 3x^{1/3} - 4$, $x + 6\sqrt{x} + 8$.

Algebraic Expressions — Factoring a sum or difference of cubes

Description. Use the identities $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ and $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$ to factor sums and differences of cubes.

Example. Factor $x^3 + 8y^3$

Solution: $x^3 + 8y^3 = (x + 2y)(x^2 - 2xy + 4y^2)$.

Samples. Factor $p^3 - 125$, $x^6 + (y + 1)^3$

Algebraic Expressions — Fractional expressions-multiplying and dividing

Description. i) To multiply fractional expressions, multiply their numerators and also their denominators. ii) To divide, multiply by the reciprocal of the divisor.

Examples. i) Multiply $\frac{x}{y+3} \cdot \frac{x+3}{y+1}$ ii) Divide $\frac{x-4}{x} \div \frac{x+1}{x-1}$

Solution. i) $\frac{x}{y+3} \cdot \frac{x+3}{y+1} = \frac{x(x+3)}{(y+3)(y+1)}$

ii) $\frac{x-4}{x} \div \frac{x+1}{x-1} = \frac{x-4}{x} \cdot \frac{x-1}{x+1} = \frac{(x-4)(x-1)}{x(x+1)}$

Algebraic Expressions — Fractional expressions- simplifying

Description. Factor the fractional expression into a product where one of the factors is equal to 1 and then "remove" that factor.

Example. Simply $\frac{x^2+x-2}{x^2-3x+2}$

Solution: $\frac{x^2+x-2}{x^2-3x+2} = \frac{(x+2)(x-1)}{(x-2)(x-1)} = \frac{x+2}{x-2} \cdot \frac{x-1}{x-1} = \frac{x+2}{x-2}$.

Samples. Simplify $\frac{4x^2y^7}{10xy^4}$, $\frac{(a^3-8)(a^2+8a+16)}{(a^2-4)}$

Algebraic Expressions — Fractional expressions - adding and subtracting

Description. When fractional expressions have the same denominator, add or subtract them by adding or subtracting the numerators and retaining the common denominator. If the denominators are not the same, first find equivalent expressions with the same denominator. The result may need to be simplified as in the previous topic.

Examples. Add or subtract as indicated: i) $\frac{3}{xy+y} + \frac{4}{x+1}$ ii) $\frac{x+5}{x^2-y^2} - \frac{y+5}{x^2-y^2}$

Solution: i) $\frac{3}{xy+y}+\frac{4}{x+1}=\frac{3}{(x+1)y}+\frac{4y}{(x+1)y}=\frac{3+4y}{(x+1)y}$

ii) $\frac{x+5}{x^2-y^2}-\frac{y+5}{x^2-y^2}=\frac{(x+5)-(y+5)}{x^2-y^2}=\frac{x-y}{x^2-y^2}=\frac{x-y}{(x+y)(x-y)}=\frac{1}{x+y}$

Samples. Add or subtract as indicated: $\frac{x}{x^2-x-20}+\frac{2}{x-5}$, $\frac{4}{x^2-x}+\frac{4}{x}-\frac{2}{x-1}$, $\frac{3a}{ax+a}-\frac{2a}{x+1}$.

Algebraic Expressions — Fractions of fractions - simplifying

Description. To simplify A/B where A and B are expressions that may contain fractions, first "clear" all denominators within A and B by multiplying A/B by C/C =1 for an appropriate expression C. Then simplify the resulting fractional expression.

Example. Simplify $\dfrac{\frac{a^2}{b^2}-\frac{b^2}{a^2}}{\frac{a}{b}-\frac{b}{a}}$

Solution:

$$\frac{\frac{a^2}{b^2}-\frac{b^2}{a^2}}{\frac{a}{b}-\frac{b}{a}}=\frac{\frac{a^2}{b^2}-\frac{b^2}{a^2}}{\frac{a}{b}-\frac{b}{a}}\cdot\frac{a^2b^2}{a^2b^2}=\frac{\frac{a^2}{b^2}a^2b^2-\frac{b^2}{a^2}a^2b^2}{\frac{a}{b}a^2b^2-\frac{b}{a}a^2b^2}=\frac{a^4-b^4}{a^3b-ab^3}$$

$$=\frac{(a^2+b^2)(a^2-b^2)}{ab(a^2-b^2)}=\frac{a^2+b^2}{ab}.$$

Samples. Simplify: $\frac{(a-b)/b}{(a^2-b^2)/(ab)}$, $\frac{x^{-1}+y^{-1}}{x^{-3}+y^{-3}}$.

Algebraic Expressions — Radical notation

Description. **i) With $\sqrt[n]{a}$ representing the (principal) nth root of a, use properties of radicals (laws of exponents for fractional exponents written in radical notation) to simplify radical expressions. ii) Rationalize denominators containing radicals.**

Examples. i) Simplify $\sqrt[3]{4(x-1)^2}\cdot\sqrt[3]{10(x-1)^2}$

ii) Rationalize the denominator $\dfrac{1}{\sqrt{x}-\sqrt{2}}$

Solution: i) $\sqrt[3]{4(x-1)^2}\cdot\sqrt[3]{10(x-1)^2} = \sqrt[3]{40(x-1)^4} = \sqrt[3]{8(x-1)^3}\cdot\sqrt[3]{5(x-1)}$

$= 2(x-1)\sqrt[3]{5(x-1)}.$

ii) $\dfrac{1}{\sqrt{x}-\sqrt{2}} = \dfrac{1}{\sqrt{x}-\sqrt{2}}\cdot\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \dfrac{\sqrt{x}+\sqrt{2}}{x-2}$

Samples. i) Simplify $\dfrac{\sqrt[3]{24x^2y^3z}}{\sqrt[3]{25z^4}}$

ii) Rationalize the denominator: $\dfrac{3}{\sqrt[3]{2x^2}}$, $\dfrac{1-\sqrt{x}}{1+\sqrt{x}}$

Algebraic Expressions — Rational exponents, definition, laws

Description. With $a^{m/n}$, $a > 0$, meaning $\left(\sqrt[n]{a}\right)^m$, i) use the laws of exponents to simply expressions with rational exponents, or ii) convert between exponential and radical notation.

Example. Simplify and write in radical notation $\dfrac{x^{2/3}y^{7/6}}{x^{-1/3}y^{1/2}}$

Solution. $\dfrac{x^{2/3}y^{7/6}}{x^{-1/3}y^{1/2}} = x^{2/3-1/3}y^{7/6-1/2} = xy^{2/3} = x\sqrt[3]{y^2}$

Samples. Simplify and write in radical notation: $16^{3/4}$, $a^{5/6}a^{2/3}$,

$\sqrt[3]{\sqrt[4]{2}}$, $\left(\dfrac{x^{2/3}}{4y^2}\right)^{-1/2}$, $\dfrac{\sqrt{(a+x)^3}\cdot\sqrt[3]{(a+x)^2}}{\sqrt[4]{a+x}}$.

Equations and Inequalities in One Unknown

Equations and Inequalities in One Unknown — General principles

Description. Use the following general principles to solve equations and inequalities, where a, b, and c are any numbers:

i) $a = b \Rightarrow a + c = b + c$
ii) $a = b \Rightarrow ac = bc$
iii) $ab = 0 \Rightarrow a = 0$ or $b = 0$
iv) $a < b \Rightarrow a + c < b + c$
v) $a < b,\ c > 0 \Rightarrow ac < bc$
vi) $a < b,\ c < 0 \Rightarrow ac > bc$.

Equations and Inequalities in One Unknown — Linear equations

Description. Use the general principles and properties of numbers to solve linear equations.

Examples. Solve $2x - 5 = 4x + 7$

Solution:

$2x - 5 = 4x + 7$
$2x - 5 - 4x + 5 = 4x + 7 - 4x + 5$
$-2x = 12$
$x = -6$
Check. The solution set is $\{-6\}$.

Equations and Inequalities in One Unknown — Quadratic equations-factoring

Description. To solve a quadratic equation by factoring, first get 0 on one side and factor the other side. Then set each of the factors equal to 0 and solve the resulting equations.

Example. Solve $2x^2 + 6x = x + 3$

Solution:

$2x^2 + 6x = x + 3$
$2x^2 + 5x - 3 = 0$
$(2x - 1)(x + 3) = 0 \Rightarrow \quad 2x - 1 = 0,$ or $x + 3 = 0$
$x = 1/2,$ or -3
Check. The solution set is $\{1/2, -3\}$.

Equations and Inequalities in One Unknown — Quadratic equations - completing the square

Description. To solve a quadratic equation by completing the square, first get an equivalent equation in the form $x^2 + Bx + C = 0$. Then "complete the square" and solve the resulting equivalent equation by "taking square roots."

Example. Solve $x^2 - 6x + 7 = 0$

Solution:

$$x^2 - 6x + 7 = 0$$
$$(x^2 - 6x + 9) + 7 = 9$$
$$(x - 3)^2 = 2 \quad \Rightarrow \quad x - 3 = \pm\sqrt{2}$$
$$x = 3 \pm \sqrt{2}$$

Check. The solution set is $\{3 + \sqrt{2},\ 3 - \sqrt{2}\}$.

Equations and Inequalities in One Unknown — Quadratic formula, discriminant

Description. (i) Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve $ax^2 + bx + c = 0$, $a \neq 0$. (ii) Use the discriminant $\sqrt{b^2 - 4ac}$ to determine the number of real roots of $ax^2 + bx + c = 0$. (iii) Determine complex roots of $ax^2 + bx + c = 0$ when the discriminant is negative.

Examples. (i) Solve $2x^2 + 1 = 4x$. (ii) For what value of a does $ax^2 - 6x + 3 = 0$ have exactly one distinct real root?
(iii) Solve $x^2 - 2x + 2 = 0$.

Solution: (i) $2x^2 - 4x + 1 = 0 \;\Rightarrow\; x = \dfrac{4 \pm \sqrt{16 - 8}}{2} = \dfrac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}.$

Check. The solution set is $\{2 + \sqrt{2},\ 2 - \sqrt{2}\}$

(ii) $ax^2 - 6x + 3 = 0$ has exactly one distinct real root when $6^2 - 4a(3) = 0$, that is, when $a = 3$.

(iii) $x^2 - 2x + 2 = 0 \;\Rightarrow\; x = \dfrac{2 \pm \sqrt{4 - 8}}{2} = \dfrac{2 \pm 2i}{2} = 1 \pm i.$

Equations and Inequalities in One Unknown — Equations with radicals

Description. To solve an equation with radicals, first isolate one of the radical terms and raise both sides of this equivalent equation to some power in order to "get rid" of this radical. Repeat this process until all radicals are elimated and then solve the resulting equation to find possible solutions. Check the possible solutions in the original equation since extraneous solutions may be introduced.

Examples. Solve $x = 3 + \sqrt{x-1}$

Solution: $x - 3 = \sqrt{x-1} \Rightarrow$

$$\begin{aligned}(x-3)^2 &= x - 1\\ x^2 - 6x + 9 &= x - 1\\ x^2 - 7x + 10 &= 0\\ (x-2)(x-5) &= 0 \Rightarrow x = 2 \text{ or } 5 \text{ are the only possibilities.}\end{aligned}$$

Check. The solution set is $\{5\}$.

Samples. Solve: $x - 5 = \sqrt{x+7}$, $\sqrt{2x-5} - \sqrt{x-3} = 1$, $\sqrt[3]{3x+5} = 2$.

Equations and Inequalities in One Unknown — Equations reducible to quadratics

Description. To solve an equation reducible to a quadratic, first make a substitution and solve the resulting equation for the new variable. Then solve for the original variable.

Example. Solve $x^4 - 5x^2 + 6 = 0$.

Solution: With $u = x^2$ the equation becomes $u^2 - 5u + 6 = 0$, so that $x^2 = u = 2$ or 3. Then $x = \pm\sqrt{2}, \pm\sqrt{3}$.

Check. The solution set is $\{\sqrt{2}, -\sqrt{2}, \sqrt{3}, -\sqrt{3}\}$

Samples. Solve: $x^{1/2} - x^{1/4} - 6 = 0$, $2x - 9\sqrt{x} + 4 = 0$, $4y^{-2} = y^{-1} + 5$.

Equations and Inequalities in One Unknown Equations with fractional expressions

Description. To solve an equation with fractional expressions, first "clear" all denominators by multiplying both sides of the equation by the "LCM" of all the denominations. Then solve the resulting equation.

Example. Solve $\frac{7}{x-9} - \frac{6}{x} = \frac{60}{x^2-9x}$.

Solution: $x(x-9)\left(\frac{7}{x-9} - \frac{6}{x}\right) = x(x-9)\left(\frac{60}{x^2-9x}\right)$.
$7x - 6(x - 9) = 60$
$x = 6$
Check. The solution set is $\{6\}$.

Samples. Solve: $\frac{1}{2} + \frac{2}{x} = \frac{1}{3} + \frac{3}{x}$, $\frac{4}{x^2-1} - \frac{2}{x-1} = \frac{3}{x+1}$, $1 - \frac{3}{x} = \frac{40}{x^2}$

Equations and Inequalities in One Unknown Linear inequalities

Description. Use the general principles and properties of numbers to solve linear inequalities.

Example. Solve $2x + 4 - 4x \leq 5(-x + 1)$.

Solution: $2x + 4 - 4x \leq 5(-x + 1)$
$-2x + 4 \leq -5x + 5$
$3x \leq 1$
$x \leq 1/3$.
Check. The solution set is $\{ x : x \leq 1/3\}$.

Equations and Inequalities in One Unknown Equations with absolute value

Description. Use $|u| = a \Leftrightarrow u = \pm a$, to solve equations with absolute value.

Example. Solve $|5x + 7| = 13$.

Solution: $5x + 7 = 13$ or $5x + 7 = -13$
$5x = 6$ or $5x = -20$
$x = 6/5$ or $x = -4$.
Check. The solution set is $\{6/5, -4\}$.

Samples. Solve: $|x - 5| = 2$, $|2x - 2| = x$, $\left|\frac{2x+1}{x}\right| = 3$.

Equations and Inequalities in One Unknown — Inequalities with absolute value

Description. Use the interpretation of absolute value of a quantity as the distance of that quantity to the origin ($|u| < a \iff -a < u < a$, $|u| > a \iff u < -a$ or $a < u$), to solve inequalities with absolute value.

Example. Solve $|3x + 2| \leq 7$.

Solution:

$$|3x + 2| \leq 7 \iff \begin{aligned} -7 &\leq 3x + 2 \leq 7 \\ -9 &\leq 3x \leq 5 \\ -3 &\leq x \leq 5/3 \end{aligned}$$

Check. The solution set is $\{ x : -3 \leq x \leq 5/3 \}$

Samples. Solve: $|2x + 1| < 5$, $|3x - 5| \geq 5$.

Equations and Inequalities in One Unknown — Quadratic inequalities

Description. To solve a quadratic inequality, first get 0 on one side and factor the other side. Use the resulting roots to divide the number line into intervals. Then determine (by the rules for multiplication of signed numbers, or the graph of the quadratic) the sign of the quadratic expression on each of the intervals. Finally, use these signs to write down the solution set of the original inequality.

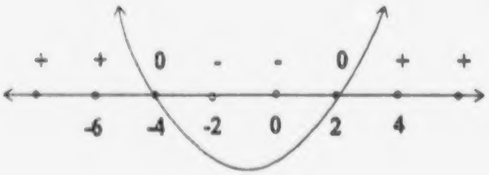
Example. Solve. $x^2 + 2x > 8$.

Solution:

$x^2 + 2x - 8 > 0$

$(x + 4)(x - 2) > 0$.

The solution set is $\{ x : x < -4$ or $x > 2 \}$.



Samples. Solve: $x^2 + 3x + 2 \leq 0$, $2x^2 + 3x > 9$, $x^2 + 6x + 2 < 0$.

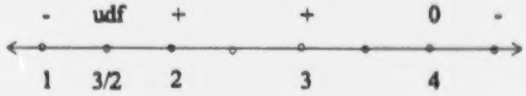
Equation and Inequalities in one unknown — Inequalities with fractional expressions

Description. To solve an inequality with fractional expressions, first add an appropriate expression to both sides of the inequality to get 0 on one side. (Do not multiply both sides by an expression as this may reverse the direction of the inequality.) Put all nonzero terms over a common denominator. Find numbers making the resulting fraction 0, numbers making the resulting fraction undefined, and numbers making the original expressions undefined. Use these numbers to divide the number line into intervals. Then determine the sign of the fraction on each of the intervals. Finally, use these signs to write down the solution set of the original inequality.

Example. Solve $\frac{x+1}{2x-3} \le 1$

Solution: $\frac{x+1}{2x-3} - 1 \le 0$ $\qquad$ $\frac{x+1}{2x-3} - \frac{2x-3}{2x-3} \le 0$

$\frac{-x+4}{2x-3} \le 0$



The solution set is { x : x < 3/2 or x ≥ 4}.

Samples. Solve: $5 \le 1/x$, $x + 9/x - 6 < 0$, $\frac{-x+1}{x-2} \ge 2$.

Problem Solving, Word Problems

Problem Solving, Word Problems General techniques

Description. Use an organized process similar to the one below to solve word problems.

Example.

1. Familiarize (read/re-read, organize information, list knowns and unknowns, assign variables, list relationships)
2. Translate (relations into equations and inequalities)
3. Solve equations and inequalities
4. Check answer
5. State answer clearly.

Problem Solving, Word Problems Ratio and proportion

Description. Set up and solve a word problem involving a proportion $\frac{a}{b} = \frac{c}{d}$.

Example. If four cans of cola cost \$1.88, how much will six cans cost?

Solution. Let x = cost of six cans of cola. From the information given

$$\frac{\text{number of cans}}{\text{dollar}} \text{ for cola} = \frac{4\text{ cans}}{\$1.88} = \frac{6\text{ cans}}{\$x} \quad \text{or}$$

$$\text{cost per can for cola} = \frac{\text{dollars}}{\text{can}} = \frac{\$1.88}{4\text{ cans}} = \frac{\$x}{6\text{ cans}}.$$

Then $x = \frac{6\text{ cans}}{4\text{ cans}} \cdot \$1.88 = \$2.82$. Check $\frac{1.88}{4} = 0.47 = \frac{2.82}{6}$.

Result, six cans of cola would cost \$2.82.

Sample. If a copying machine can make 33 copies in 4 minutes, how long will it take to make 44 copies?

Problem Solving, Word Problems Percent

Description. Using i/100 for i%, set up and solve problems involving percentages, percent increases, and percent decreases.

Example. A person received a 12% raise that is \$2520. What were the old and new salaries?

Solution. Let S and N denote the old and new salaries, respectively. The problem states that $0.12S = \$2520$. Then, $S = \$21{,}000$, and $N = S + \text{raise} = \$21{,}000 + \$2{,}520 = \$23{,}520$. Check 12% of $\$21{,}000 = 0.12(21000) = 2520$. Result, the old salary was \$21,000 and the new salary was \$23,520.

Samples. 1. The Smiths have a montly income of \$2350 and a monthly mortgage payment of \$940. What percentage of their montly income is their mortgage payment?
2. Bob is three years older than Amy. Four years ago, Bob was 50% older than Amy. What are their present ages?
3. The gross sales for Abbot's Widgets for 1992 was \$216,000 which represents a 10% decrease from the gross sales for 1991. What was Abbot's gross sales for 1991?

Problem Solving, Word Problems Interest

Description. (i) Use the formulas $I = Prt$ and $A = P + I$ to set up and solve simple interest problems. (ii)Use the formulas $A = P(1 + r/n)^{nt}$ and $I = A - P$ to set up and solve compound interest problems.
Here P denotes the present value or principal invested, r denotes the annual percentage rate expressed as a decimal, t denotes the number of years, A denotes the future value or combined interest and principal, and n denotes the number of times per year that interest is compounded.

Examples. (i) How much interest would one earn in three years with 8% simple interest on an initial investment of \$73 assuming that interest is withdrawn each year?
(ii) What is the value in 9 months of \$1000 invested at 8% compounded quarterly? What is the total interest earned?

Solution. (i) $I = Prt = \$73 \times 0.08 \times 3 = \$17.52$
(i) $A = P(1 + r/n)^{nt} = \$1000(1 + .08/4)^{4(3/4)} = \$1000(1.02)^3 = \$10{,}612.08$. $I = A - P = \$612.08$.

Problem Solving, Word Problems Travel

Description. Use distance = speed × time, and additive properties of speed to set up and solve travel problems.

Example. A boat travels 50 km downstream in the same time that it takes to go 30 km upstream. The speed of the boat in still water is 16 km/hr. Find the speed of the current of the stream.

Solution. Let c denote the speed of the current and t be the time needed to go 30 km upstream. In table form we have:

	distance	speed	time
downstream	50	$16 + c$	t
upstream	30	$16 - c$	t.

We can write two equations $50 = (16 + c)t$ and $30 = (16 - c)t$. Solving each of these for t and equating the results gives $\frac{50}{16+c} = \frac{30}{16-c}$ which yields $c = 4$ km/hr for the speed of the current.

Problem Solving, Word Problems Geomtric formulas

Description. Set up and solve problems involving elementary geomtric formulas including perimeters of triangles and rectangles; circumferences of circles; areas of triangles, rectangles and circles; volumes of rectangular boxes (parallelopipeds); and sides of right triangles.

Example. The base of a rectangle is 50% longer than its heigth. Find the dimensions of the rectangle if the perimeter is 20 cm.

Solution. Let b denote the length of the base and let h denote the heigth. The problem states that $b = h + 0.5h = 1.5h$, and that $2b + 2h = 20$ cm. Substituting for b in the expression for the perimeter gives $2(1.5h) + 2h = 20$, or $5h = 20$. Then $h = 4$cm and $b = 1.5(4 \text{ cm}) = 6$ cm.

Sample. An open top box is made from a 10 cm by 20 cm piece of cardboard by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm.^{2} What is the length of the sides of the squares cut from the corners?

Problem Solving. Word Problems — Geometry of lines and similar triangles

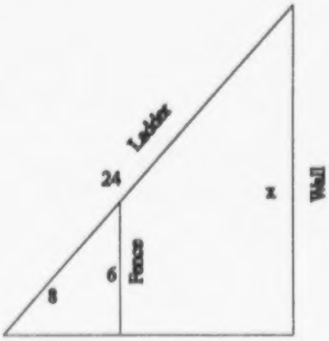
Description. Set up and solve problems using proportions based on similar triangles. The similar triangles may be determined by properties of parallel and perpendicular lines.

Example. A 24 foot ladder is leaned over a 6 foot high fence and rests against a wall. The ladder just touches the top of the fence, and this point is 8 feet from the base of the ladder. How high is the point on the wall where the ladder rests?

Solution. First draw the diagram and identify the relevant parts. From the similar triangles set up the proportion

$$\frac{x}{6} = \frac{24}{8}$$ which gives $x = 18$.

Then the ladder rests against the wall 18 feet above the base of the wall.



Probelm Solving. Word Problems — Mixtures

Description. Set up and solve problems involving uniform mixtures of two substances.

Example. Beer A is 6% alcohol and Beer B is 2% alcohol. How many liters of each should be mixed in order to get a mixture of 50 liters that is 3.2% alcohol?

Solution. Let x and y denote the number of liters of Beer A and Beer B respectively. To obtain a total of 50 liters we must have $x + y = 50$ (E1). The total amount of alcohol in the mixture is to be $.06x + .02y = (.032)50$ liters or equivalently $6x + 2y = 160$ (E2). From (E1) $y = 50 - x$ which may be substituted into (E2) yielding $6x + 2(50 - x) = 160$. Solving these equations gives $x = 15$ liters of Beer A, and $y = 35$ liters of Beer B.

Sample. A full automobile radiator with a capacity of 16 liters, contains a mixture of antifreeze and water that is 30% antifreeze. How much of this mixture must be drained and replaced by pure antifreeze, in order to obtain a resulting mixture of 50% antifreeze in the radiator?

Problem Solving, Word Problems — Averages, means

Description. Use average value defined as the sum of values divided by the number of values to set up and solve problems involving averages or means.

Example. A construction crew has two members making \$16.50 per hour, and one member making \$18.00 per hour. If the crew has four members and the average hourly wage for the crew is \$17.50, what is the hourly wage of the fourth member?

Solution. Let W be the hourly wage of the fourth member. Then the \$17.50 average must be obtained from $(16.5 + 16.5 + 18 + W)/4 = 17.5$. Solving for W we get $W = \$19.00$ for the hourly wage of the fourth worker.

Problem Solving, Word Problems — Units of measurement

Description. Convert between different units of measurement such as minutes to hours or centimeters to meters, and use units of measurement as a check for correct use of formulas.

Examples. (i) Convert 1.5 days to minutes, convert 80 cm/sec to m/min, and convert 1440 in^2 to ft^2.
(ii) How far does a train travel at 40 miles per hour in 3 hours? Use units to check your calculation.

Solution. (i) $1.5 \text{ days} = (1.5\text{days})\left(\frac{24\text{hrs}}{1\text{day}}\right)\left(\frac{60\text{ min}}{1\text{hr}}\right) = 2160 \text{ min.}$

$$80\left(\frac{\text{cm}}{\text{sec}}\right) = \left(\frac{80\text{cm}}{1\text{sec}}\right)\left(\frac{1\text{m}}{100\text{cm}}\right)\left(\frac{60\text{sec}}{1\text{min}}\right) = 48\left(\frac{\text{m}}{\text{min}}\right).$$

$$1440 \text{ in}^2 = (1440 \text{ in}^2)\left(\frac{1\text{ft}}{12\text{in}}\right)^2 = 10 \text{ ft}^2.$$

(ii) Distance = speed × time = $\left(\frac{40\text{miles}}{\text{hr}}\right)(3 \text{ hr}) = 120$ miles, and miles is a correct unit for distance.

Description. Set up and solve problems involving direct, indirect and other types of variation.

Examples. (i) By Hooke's Law the distance a spring stretches varies directly with the amount of weight attached. The constant of this variation is called the spring constant. If a spring is stretched 5 inches by a 10 lb. weight, find the spring constant, and the weight needed to stretch the spring 15 inches.

(ii) The volume of an enclosed gas varies inversely as the pressure on the gas. If a contractible cylinder contains10,000 ft^3 of gas under a pressure of 20 lb/ft^2, and the cylinder is contracted until a pressure of 25 lb/ft^3 is reached, what is the resulting volume.

(iii) The resistance of a wire to the flow of electricity varies directly as its length and inversely as the square of its diameter. If a wire 36 inches long and 0.5 inches in diameter has a resistance of 1.44 Ohms, find the length of a wire made of the same material whose resistance is 0.8 Ohms and whose diameter is 0.6 inches.

Solutions. (i) The variation is described by $D = kW$ where D is the distance stretched, k is the spring constant, and W is the weight. First, we are given that $5 \text{ in} = k(10 \text{ lb})$ so the spring constant is $k = 0.5$ (in/lb). Second, solve $15 \text{ in} = 0.5 \text{ (in/lb) } W$ to find that a 30 pound weight is needed to stretch the spring 15 inches.

(ii) The variation is described by $V = k/P$ where V is the volume, k is the constant of variation, and P is the pressure. First, $10{,}000 \text{ ft}^3 = k/(20 \text{ lb/ft}^2)$, so that $k = 200{,}000$ ft.lb. Second, if P is increased to 25 lb/ft^2, then $V = 200{,}000 \text{ ftlb}/(25 \text{ lb/ft}^2) = 8000 \text{ ft}^3$.

(iii) We have $R = kL/(d^2)$ where R is the resistance, L is the length, d is the diameter, and k is some constant. First, $1.44 = 36k/(0.5)^2$ so that $k = .01$. Second, solve $0.8 = .01L/(0.6)^2$ for L to obtain a length $L = 28.8$ inches.

Functions

Functions — **Cartesian Products**

Description. Use the definitions of ordered pair and Cartesian product to find the Cartesian product A × B of two finite sets A and B.

Example. If $A = \{a, b, c\}$ and $B = \{5, 7\}$, what is $A \times B$?

Solution: $A \times B = \{ (a,5), (a,7), (b,5), (b,7), (c,5), (c,7) \}$

Samples. If $A = \{-1, 0, 4\}$ and $B = \{d, k\}$, what are $A \times B$ and $B \times C$?
If $C = \{0, 2, 6)$ what is $C \times C$?

Functions — **Relations, domain, range**

Description. Use the definition of relation to specify certain correspondences from a set A to a set B. Also give the domain and range of relations.

Example. If A is the set of students {Mary, John, Joe, Susan, Ann} and B is the set of ages {19,20,21,22}, give the relation from A to B which indicates the age of each boy. Also give the domain and range of this relation.

Solution: Depending on the ages, the relation might be, for example, {(John, 22), (Joe, 19)}. The domain is {John, Joe} and the range is {19,22}.

Sample: If $A = \{0,1,2,3\}$ find the subset of $A \times A$ which gives the ordered pairs in the > relation.

Functions — **Recognizing functions; domain, range**

Description. Use the definition of function to determine whether or not a given relation from a set A to a set B is a function. Also give the domain and range of functions.

Example. Determine whether each of the sets
$H = \{(2,6),(-1,4),(0,6),(4,5)\}$ and $K = \{(1,2),(5,7),(1,8)\}$ is a function.
In any case, determine the domain and range of each relation.

Solution: The relation H is a function since no two ordered pairs have the same first coordinate and different second coordinates. The relation K is not a function since the ordered pairs *(1,2)* and *(1,8)* have the same first coordinate but different second coordinates.

The domains of H and K are {2,-1,0,4} and {1,5}, respectively, while their ranges are {6,4,5} and {2,7,8}.

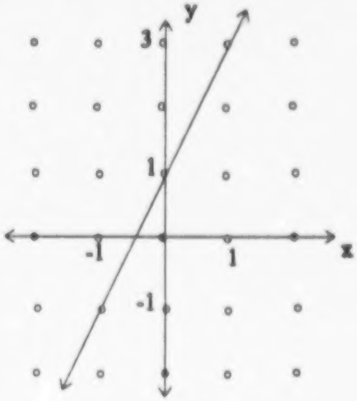
Sample. Is the set $L = \{(3,3), (-1,3), (7,-1)\}$ a function? Given the domain and range of this relation.

Description. By making general considerations and plotting points on a Cartesian coordinate system graph relations which are subsets of $R \times R$ determined by solutions sets of equations.

Example. Graph $y = 2x + 1$.

Solution. We make a table of ordered pairs of some solutions of $y = 2x + 1$ and plot the corresponding points. To obtain the graph we make a drawing which contains the points.

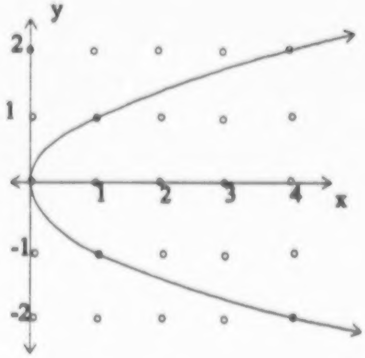
x	y
-1	-1
0	1
1	3



Example. Graph $x = y^2$.

Solution: We make a table and then the drawing. Here we can first select numbers for y and then find the corresponding x-values.

x	y
4	-2
4	2
1	-1
1	1
0	0



Samples. Graph: $y = -3x + 2$, $y = x^2 + 1$, $y = 5 - x^2$, $y = |x|$, $xy = 4$, $x = y^2 - 2$, $x = |y - 1|$.

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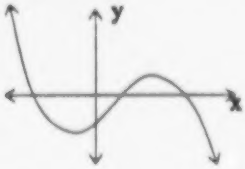
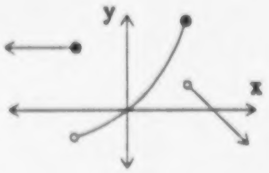
Functions — Vertical line test

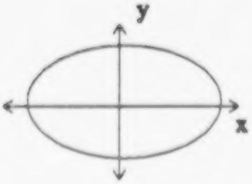
Description. Use the vertical line test to determine whether a graph is the graph of a function.

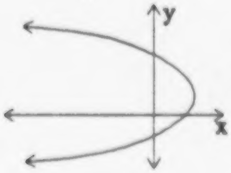
Examples.





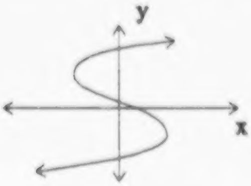
are each examples of functions since no vertical line intersects either graph at more than one point.

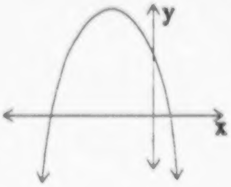


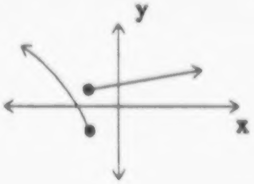


are not the graphs of functions.

Samples. Determine whether each graph is the graph of a function:

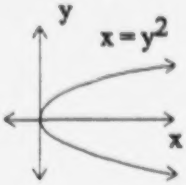


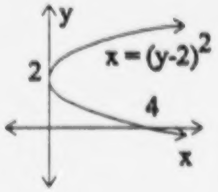




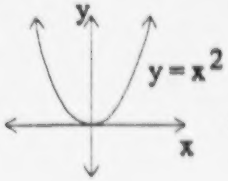
Description. Graph relations in (x - h) and (y - k) by a vertical and/or horizontal "shift."

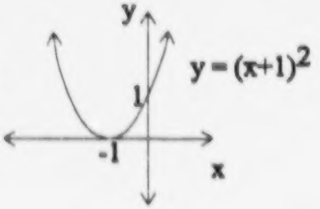
Examples. Shift the graph of $x = y^2$ up 2 units to obtain the graph of $x = (y-2)^2$:



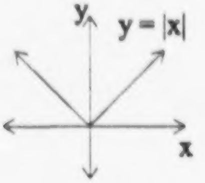


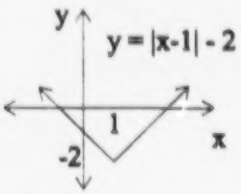
Shift the graph of $y = x^2$ to the left 1 units to obtain the graph of $y = (x + 1)^2$:





Shift the graph of $y = |x|$ to the right 1 unit and down 2 units to obtain the graph of $y = |x - 1| - 2$:





Samples. Using shift of axes sketch the graphs of: $y = x^2 - 3$, $x = y^2 + 1$, $y = -|x| + 2$, $y = |x + 4| - 1$, $y = 2x^2 + 4$, $x = |2y| - 1$, $y = |x + 5| + 3$.

Functions Notation for functions, function values,and difference quotient

Description. Use an f(x) type notation to find function values and to evaluate functions such as in difference quotients.

Example. If $f(x) = 3x^2 + 1$, find $f(-1)$, $f(c+2)$, and the difference quotient $\frac{f(a+h)-f(a)}{h}$, and simplify.

Solution: $f(-1) = 3(-1)^2 + 1 = 4$.
$f(c+2) = 3(c+2)^2 + 1 = 3(c^2 + 4c + 4) + 1 = 3c^2 + 12c + 13$.

$$\frac{f(a+h)-f(a)}{h} = \frac{3(a+h)^2+1-(3a^2+1)}{h} = \frac{6ah+3h^2}{h} = 6a+3h$$

Samples. For each of the following functions, find $f(-2)$,$f(2)$,$f(t-1)$, and simplify:
$f(x) = 6x - 1$, $f(x) = -3x + 4$, $f(x) = 2x^2 + x$, $f(x) = 2|x| + 5$.

Functions Algebra of functions

Description. Using the definitions, find the sum difference, product, and quotient of two functions.

Example. If $f(x) = 2x + 1$ and $g(x) = x^2 - 4$, find the functions $f + g$, $f - g$, fg, f/g and simplify.

Solution: $(f + g)(x) = f(x) + g(x) = 2x + 1 + x^2 - 4 = x^2 + 2x - 3$.

$(f - g)(x) = f(x) - g(x) = 2x + 1 - (x^2 - 4) = -x^2 + 2x + 5$.

$(fg)(x) = f(x)g(x) = (2x + 1)(x^2 - 4) = 2x^3 + x^2 - 8x - 4$.

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{2x+1}{x^2-4}$$

Samples. For each of the following pairs of functions, find the functions
$f + g$, $f - g$, $g - f$, fg, ff, g/f and simplify:

$f(x) = x^2 - 2$, $g(x) = 3x + 1$

$f(x) = 7x + 2$, $g(x) = x^3 - 2$

$f(x) = \sqrt{x}$ $g(x) = 4x + 3$.

Functions Composition

Description. Using the definition, find the composition of two functions.

Examples. For each of the following pairs of functions, find the functions fog, gof, and simplify:

i) $f(x) = x^2 - 3$, $g(x) = 2x + 1$

ii) $f(x) = \sqrt{x+4}$, $g(x) = 3x - 7$.

Solutions:

i) $f\circ g(x) = f(g(x)) = (2x + 1)^2 - 3 = 4x^2 + 4x - 2$

$g\circ f(x) = g(f(x)) = 2(x^2 - 3) + 1 = 2x^2 - 5$

ii) $f\circ g(x) = f(g(x)) = \sqrt{(3x-7)+4} = \sqrt{3x-3}$

$g\circ f(x) = g(f(x)) = 3\sqrt{x+4} - 7$.

Samples. For each of the following pairs of functions, find the functions f o g, g o f, and simplify:

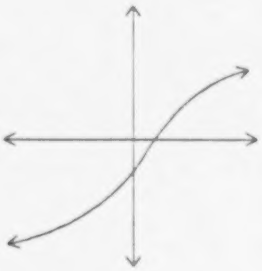
$f(x) = 4x - 1$, $g(x) = x^2 + 2$; $f(x) = 2x + 3$, $g(x) = \sqrt{x^2-4}$;

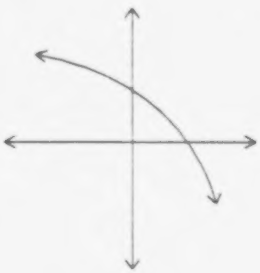
$f(x) = \dfrac{1}{1+x}$, $g(x) = \dfrac{1-x}{x}$; $f(x) = \sqrt[3]{x}$, $g(x) = x^3$.

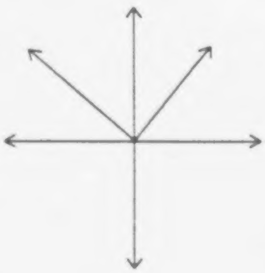
Functions Increasing/decreasing functions

Description. Using the definitions and general shape of the graph, determine whether a function is increasing, decreasing, or neither.

Examples.

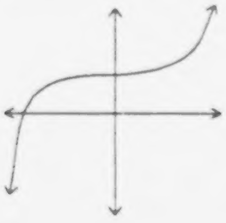


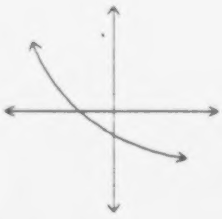


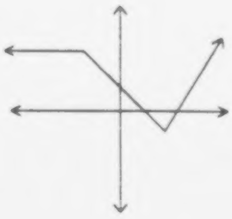


Solution. Increasing Decreasing Neither.

Samples. Determine whether each graph is the graph of an increasing function, a decreasing function, or neither.





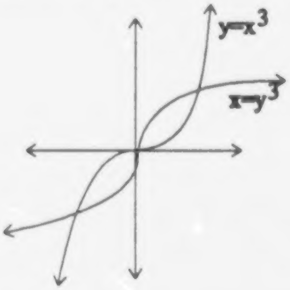


Functions

Inverses of relations (and functions)

Description. Use the definition to find the inverse of a relation and reflect the graph of the relation about the line $y = x$ to obtain the graph of the inverse.

Examples. The inverse of the relation $\{(2,1),(-4,0),(5,7),(2,9)\}$ is the relation $\{(1,2),(0,-4),(7,5),(9,2)\}$.
The inverse of the relation $y = 2x - 6$ is $x = 2y - 6$.
Graphs of $y = x^3$ and the inverse $x = y^3$:



Samples. Find the inverse of the relations defined by: $\{(4,2),(0,1),(-1,2)(3,8)\}$, $4x^2 + 9y^2 = 18$, $xy = -4$, $y = x^2 + 1$.
Graph each of the following relations and its inverse on a Cartesian coordinate system: $y = 2x + 1$, $x = -y^2$, $y = |x| + 2$.

Functions

Symmetry About the line y = x

Description. Determine whether the graph of a relation is symmetric with respect to the line $y = x$, that is, determine whether a relation and its inverse are the same.

Examples. Determine whether the graph of each of the relations:
$H = \{(1,4),(0,8),(8,0),(4,1)\}$, $y = x^2 + 3$, and $2x + 2y = 5$
is symmetric with respect to the line $y = x$.

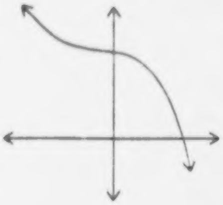
Solution: The inverse of H is $\{(4,1),(8,0),(0,8),(1,4)\}$, which is the same as H.
The inverse of $y = x^2 + 3$ is $x = y^2 + 3$, which is not the same as the original relation.
Thirdly, the relation $2x + 2y = 5$ and its inverse $2y + 2x = 5$ are the same.
Thus, the graphs of each of the first and third relations are symmetric with respect to the line $y = x$.

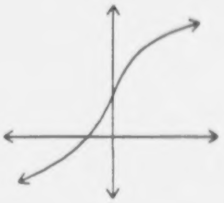
Samples. Determine whether the graph of each relation is symmetric with respect to the line $y = x$: $\{(6,2),(-2,5),(2,6),(8,4),(5,-2)\}$, $x\,y = 1$, $y = -x + 2$, $4x + 4y^2 = 7$, $y = 2/x$.

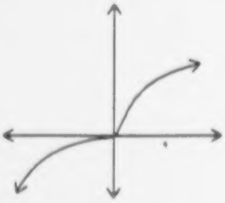
Functions — One-to-One functions, horizontal line test

Description. Use the horizontal line test to determine whether a function is one-to-one.

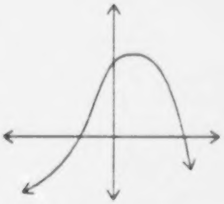
Examples.

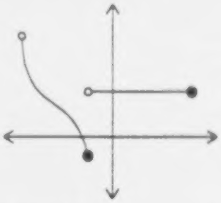






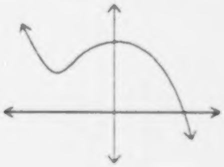
are each graphs of one-to-one functions since no horizontal line intersects either graph at more than one point, while the graphs

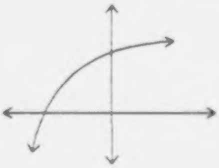


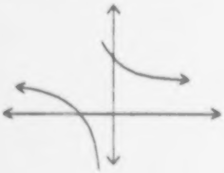


are not graphs of one-to-one functions.

Samples. Determine whether each graph is the graph of a one-to-one function:







Functions — Formula for an inverse

Description. Given an algebraic expression in x defining a one-to-one function find an algebraic expression defining the inverse function $y = f^{-1}(x)$.

Example. Find an expression for the inverse of the function $f(x) = (2x - 3)/x$.

Solution: In the equation $y = (2x - 3)/x$ interchange x and y to obtain $x = (2y - 3)/y$. Then solve for y to obtain $y = -3/(x-2)$ and $f^{-1}(x) = -3/(x-2)$.

Samples.

1. Find a formula for the inverse of $f(x) = 3x + 4$ and $g(x) = \frac{2x-3}{x+4}$.
2. Find a formula for the inverse of $(x - 1)/x$. Check your result by finding $f \circ f^{-1}$ and $f^{-1} \circ f$.

Functions — Even and odd functions

Description. Determine whether a function is even [$f(-x) = f(x)$ for all x in the domain], odd [$f(-x) = -f(x)$ for all x in the domain], or neither.

Examples. Determine whether each of the following functions is even, odd or neither: $f(x) = x^2 - 4|x| + 5$, $g(x) = 3x - 4/x$, $h(x) = x^3 + x^2$.

Solution. $f(-x) = (-x)^2 - 4|-x| + 5 = x^2 - 4|x| + 5 = f(x)$, so f(x) is even.
$g(-x) = 3(-x) - 4/(-x) = -3x + 4/x = -(3x - 4/x) = -g(x)$, so g(x) is odd.
$h(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$ is neither h(x) nor $-h(x) = -x^3 - x^2$, so h(x) is neither even nor odd.

Equations in Two Variables

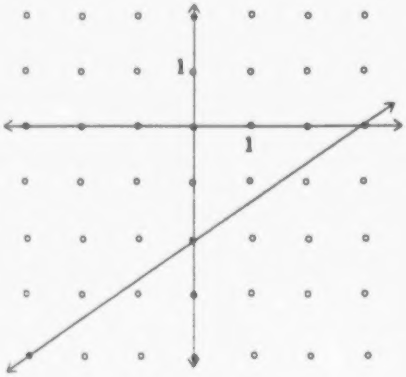
Equations in Two Variables — Linear equations-graphing, slope

Description. By finding the intercepts and another point on the line, graph a linear equation; given two points on a line use the formula to find its slope.

Example. Graph $2x - 3y = 6$.

Solution:

x	y
0	-2
3	0
-3	-4



Example. Find the slope m of the line containing the points (-3, 12), and (4, 9).

Solution: $m = \frac{9-12}{4-(-3)} = -\frac{3}{7}$

Samples. Graph the lines: $-4x + 5y = 20$, $y = -6$, $x = 1$, $x = 2 - y$.
Find the slope of the line containing the points (2, -6) and (-3, 4).

Equations in Two Variables — Lines-point slope equation

Description. Given the slope of a line (or two points on the line) and a point on the line, use the formula to find the point-slope form of the line.

Example. Find an equation of the line with slope -2 and containing the point (-4,1).

Solution: Substitute in $y - y_1 = m(x - x_1)$ to obtain $y - 1 = -2(x + 4)$ or $y = -2x - 7$.

Samples. Find an equation of the line with slope 2/3 and containing the point (1,-5).
Find an equation of the line containing the points (-1, 3) and (2,-2).

Equations in Two Variables Lines-slope intercept equation

Description. (i) Given the slope and y-intercept of a line, use the formula to find the slope-intercept form of the line. (ii) Transform a linear equation into slope-intercept form, and identify the slope.

Examples. (i) Find an equation of the line with slope -4 and y-intercept 9.
(ii) Transform the linear equation $2x - 3y = 6$ into slope-intercept form, and identify the slope.

Solution: (i) Substitute in $y = mx + b$ to obtain $y = -4x + 9$.
(ii) Solve $2x - 3y = 6$ for y to obtain $y = (2/3)x - 2$. The slope is $2/3$.

Samples. Find an equation of the line with slope 7 and y-intercept -5. Transform the linear equation $-4x + 3y = 8$ into slope-intercept form, and identify the slope.

Equations in two variables Parallel and Perpendicular

Description. (i) Use the fact that two nonvertical lines with slopes m_1, m_2 are parallel (perpendicular) if and only if $m_1 = m_2$ $(m_1 m_2 = -1)$ to determine if given lines are parallel (perpendicular). (ii) Also use these facts to find equations of lines.

Examples. (i) Determine if the lines $L_1: x + 3y = 9$ and $L_2: 2x + 6y = 12$ are parallel.
(ii) Find an equation of the line L which is perpendicular to the line $L_1: y = 2x + 1$ and contains the point $(-3, 4)$.

Solution: (i) $x + 3y = 9$ gives $y = (-1/3)x + 3$, while $2x + 6y = 12$ becomes $y = (-1/3)x + 2$. Since the slopes of L_1 and L_2 are each $-1/3$, they are parallel.
(ii) The slope of L_1 is 2 so the slope of a perpendicular line is $-1/2$. An equation for L is then $y - 4 = (-1/2)(x + 3)$ or $y = (-1/2)x + 5/2$.

Functions Distance formula in the plane

Description. Use the formula $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ to find the distance between two points (x_1, y_1) and (x_2, y_2).

Example. Find the distance between the points $(2, 1)$ and $(6, 4)$.

Solution. $D = \sqrt{(2-6)^2 + (1-4)^2} = 5$.

Equations in Two Variables

Midpoints of line segments

Description. Given the endpoints of a line segment, use the formula to find the midpoint of the segment.

Example. Find the midpoint of the line segment with endpoints (4,-7) and (1,3).

Solution: Subsituting in $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, we obtain the midpoint (5/2, -2).

Sample. Find the midpoint of the line segment with endpoints (-8,2) and (6,-9).

Equations in Two Variables

Equations of Circles

Description. (i) Given the center and radius, use the standard form to find an equation of the circle. (ii) By completing squares, transform the equation of a circle into standard form; then give the center and radius of the circle.

Examples. (i) Find an equation of the circle with center (3,-1) and radius 2.
(ii) Write in standard form, and find the center and radius of the circle $x^2 + 5x + y^2 - 4y = 1$.

Solution: (i) Substituting in $(x - h)^2 + (y - k)^2 = r^2$ gives $(x-3)^2 + (y+1)^2 = 4$.
(ii) The equation becomes $x^2 + 5x + 25/4 + y^2 - 4y + 4 = 1 + 25/4 + 4$ or $(x + 5/2)^2 + (y - 2)^2 = 21/2$. Hence, the center is (-5/2, 2), and the radius is $\sqrt{21/2} = \sqrt{42}/2$.

Samples. (i) Find an equation of the circle with center (-5,6) and radius $\sqrt{7}$.
(ii) Find the center and radius of the circle $x^2 - 8x + y^2 + 3x = -7$.

Equations in Two Variables — Parabolas

Description. By completing a square transform the equation of a parabola $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$. Then write down the vertex, line of symmetry, and max or min of the parabola. Using this information, possible intercepts, and other points, graph the parabola.

Example. Find the vertex, line of symmetry, and max or min of the parabola $y = 2x^2 + 12x + 14$. Graph the parabola.

Solution:

$y = 2(x^2 + 6x) + 14$

$y = 2(x^2 + 6x + 9 - 9) + 14 = 2(x^2 + 6x + 9) - 18 + 4$

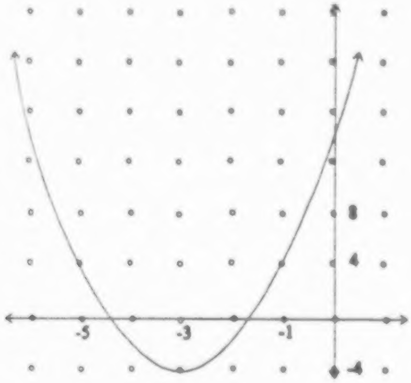
$y = 2(x + 3)^2 - 4$

Hence, the vertex is $(-3,-4)$ and the line of symmetry is the line $x = -3$.

Since $a = 2$ is positive, -4 is the minimum y - value (and the curve is concave up).

To find the x-intercepts set $y = 0$, and solve $2x^2 + 12x + 14 = 0$ for x.

We obtain $x = \dfrac{-6 \pm \sqrt{36 - 28}}{2} = -3 \pm 2\sqrt{2} \approx -4.4$ and -1.6



Samples. For each of the following parabolas, find the vertex, line of symmetry, max or min, and sketch the graph: $y = x^2 + 4x$, $y = -x^2 + 8x - 7$, $y = 3x^2 - 24x + 50$, $y = -2x^2 + 8x - 1$, $y = (-1/2)x^2 - 3x + 1/2$.

Description. Set up and solve word problems involving quadratic functions and equations.

Examples. (i) Find the points on the line $y = 6 - x$ which are 5 units distant from the point $(6, 1)$.
(ii) Find two numbers whose sum is 20 and whose product is a maximum.

Solution. (i) The coordinates of the points have the form $(x, 6 - x)$ since they are on the line. Squaring the distance formula then gives $25 = (x - 6)^2 + (6 - x - 1)^2$. This quadratic has two solutions, $x = 2$ and 9.
The points are then $(2, 4)$ and $(9, -3)$.
(ii) If x is one of the numbers, then the other number is $20 - x$. The product is $P = x(20 - x) = 100 - (x - 10)^2$. This has a maximum when $x = 10$. The two numbers are then 10, and $20 - 10 = 10$.

Samples. (i) A projectile is shot upward with an initial velocity of 8 feet per second from a platform 20 feet above the gound. What is the maximum heigth above the ground achieved by the projectile and how long is it before the projectile hits the ground?
(ii) A \$10,000 investment made in a six month CD that compounds interest quarterly, returns \$10, 609 after six months. What is the annual interest rate paid by the CD?

Exponential and Log Functions

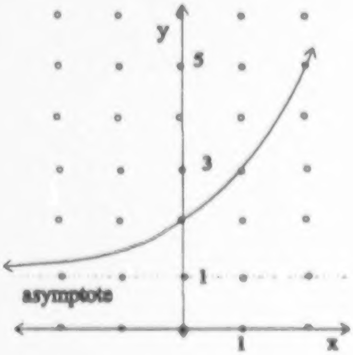
Exponential and Log Functions — Graphs of exponential functions

Description. By using continuity, monotone properties, and plotting points sketch the graphs of exponential functions.

Example. Graph $y = 2^x + 1$.

Solution

x	y
-2	5/4
-1	3/2
0	2
1	3
2	5



Samples. Graph: $y = 2^x$, $y = 2^{-x}$, $y = 3^x - 4$, $y = 2^{(x-1)} + 2$, $y = (1/3)^x$.

Exponential and Log Functions — Log functions as inverses of exponential functions

Description. i) Express the inverse of an exponential function as a logarithmic (log) function, ii) make a table for a log function by reversing the ordered pairs of the corresponding exponential function, and iii) graph a log function by reflection about the line $y = x$ (check by plotting points).

Example. i) Give the inverse of the function $f(x) = 2^x$, ii) make a table of values for the inverse function, and iii) sketch its graph.

Solution. i) $f^{-1}(x) = \log_2 x$,

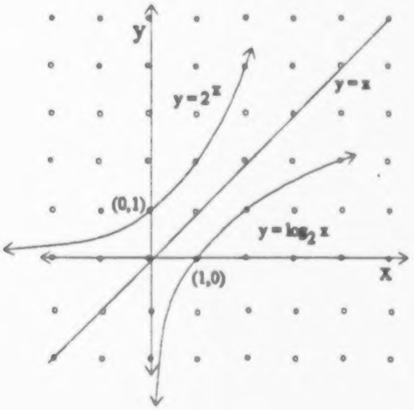
ii)

x	2^x
-2	1/4
-1	1/2
0	1
1	2
2	4

⇒

x	$\log_2 x$
1/4	-2
1/2	-1
1	0
2	1
4	2

iii)



Samples. For each of the following functions: i) Give the inverse of the function, ii) make a table of values for the inverse function, and iii) sketch its graph. $f(x) = (1.5)^x$, $g(x) = 3^x$.

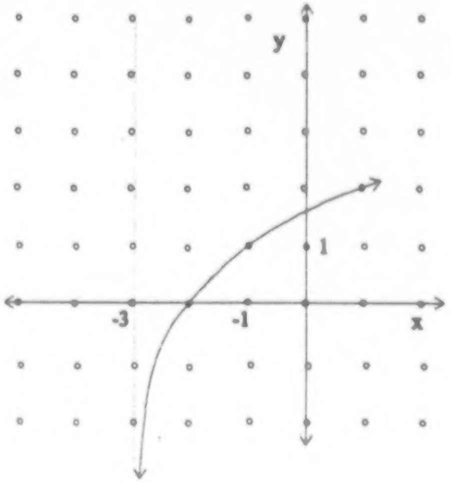
Exponential and Log Functions Graphs of log functions

Description. Graph a log function using the general form of a log function, and plotting points obtained from the corresponding exponential relation. Shift of axes can be used as appropriate.

Example. Graph $y = f(x) = \log_2 (x + 3)$.

Solution. $y = \log_2 (x + 3)$ is equivalent to $x + 3 = 2^y$ or $x = 2^y - 3$. Points (x, y) on the graph of $y = f(x)$ can be obtained by substituting y-values in $x = 2^y - 3$, and then calculating the corresponding x-values.

Alternatively, the graph of $y = \log_2 x$ could be shifted left 3 units.



Samples. Graph: $y = \log_3 x$, $y = 1 - \log_{0.5} x$, $y = \log_2 (x - 2)$, $y = \log_2 (1 - x)$.

Exponential and Log Functions Domain and range

Description. Using knowledge of the domains and ranges of the functions a^x and $\log_a x$, and properties of axes shifts, determine the domains and ranges of exponential and logarithmic functions.

Example. Determine the domain and range of $f(x) = 2 + 1.5^x$, and $g(x) = \log_2 (x + 3)$.

Solution. Since the domain of 1.5^x is all real numbers, the domain of f(x) is also all real numbers. Since the range of 1.5^x is $\{ y : y > 0 \}$, the range of f(x) is $\{ y : y > 2 \}$. Since the domain of $\log_2 u$ is $\{ u : u > 0 \}$, the domain of g(x) is $\{ x : x > -3 \}$. The range of g(x) is all real numbers.

Samples. Find the domains and ranges of the functions $f(x) = 3 - 2^{(1 - x)}$, and $g(x) = 1 + \log_2 (x - 5)$.

Exponential and Log Functions — Conversion: log ↔ exp

Description. Using the equivalence of $b = \log_a c$ and $a^b = c$, convert from logarithmic equations to exponential equations, and vice-versa.

Example. Convert: $3 = \log_4 64$ to exponential equation
$6^x = 1/36$ to logarithmic equation

Solution. $3 = \log_4 64 \quad \Leftrightarrow \quad 4^3 = 64$
$6^x = 1/36 \quad \Leftrightarrow \quad \log_6 (1/36) = x$

Samples. Convert to corresponding (exp. or log) form:
$\log_2 16 = 4$, $\log_{10} (x-1) = -1$, $\log_3 1 = t - 4$,
$7^1 = 7$, $8^{(1-x)} = 2$, $10^{-2} = s$.

Exponential and Logarithmic Functions — Properties of logs

Description. Using $\log MN = \log M + \log N$, $\log M/N = \log M - \log N$, and $\log M^p = p \log M$, find equivalent logarithmic expressions.

Examples. i) Express $\log_a \sqrt[4]{\dfrac{x^2 z^3}{y^8}}$ in terms of logs of x, y, and z individually.

ii) Write $2 \log_a y - 3 \log_a z + \log_a x$ as a single log.

Solution. i) $\log_a \sqrt[4]{\dfrac{x^2 z^3}{y^8}} = \dfrac{1}{4} \log_a \dfrac{x^2 z^3}{y^8} = \dfrac{1}{4}\left(\log_a x^2 + \log_a z^3 - \log_a y^8\right)$

$= (1/4)(2 \log_a x + 3 \log_a z - 8 \log_a y)$
$= (1/2) \log_a x + (3/4) \log_a z - 2 \log_a y$

ii) $2 \log_a y - 3 \log_a z + \log_a x = \log_a y^2 - \log_a z^3 + \log_a x$

$= \log_a \dfrac{y^2 x}{z^3}$

Samples. i) Express $\log_a \dfrac{xy^2}{a\sqrt{z}}$ in terms of logs of x, y, and z individually.

ii) Write $(1/3)\log_a y - 2 \log_a x + (1/2)\log_a z$ as a single log.

Exponential and Log Functions — Change of base

Description. Find logarithms by using $\log_a x = \dfrac{\log_b x}{\log_b a}$ and a calculator.

Example. $\log_2 34 = \dfrac{\log_{10} 34}{\log_{10} 2} \approx \dfrac{1.5315}{.3010} \approx 5.088$

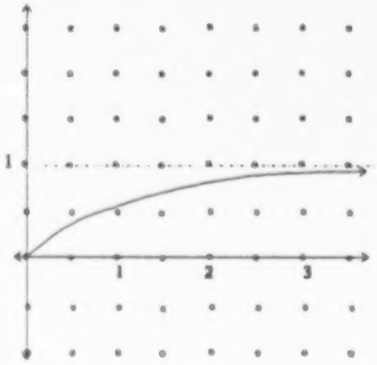
Exponential and Log Functions — Natural logs

Description. Graph base e exponential and logarithmic functions using a calculator (or a few given values $e \approx 2.7$, $1/e \approx 0.37$), and the general shapes of exponential and logarithmic curves.

Examples. Graph: i) $y = 1 - e^{-0.8x}$, $x \geq 0$ ii) $y = \ln(x + 1)$. [ln denotes $\log_e$]

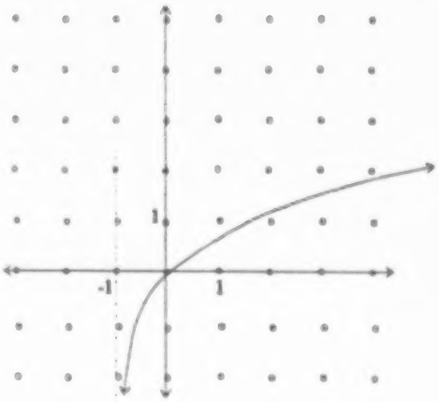
Solution. i)

x	$e^{-.8x}$	$1-e^{-.8x}$
0	1	0
0.5	.67	.33
1	.45	.55
2	.20	.80



ii)

x	x+1	ln(x+1)
-.63	.37	-1
0	1	0
1.7	2.7	1



Samples. Graph: $y = e^{-x}$, $y = e^{(x+2)}$,
$y = 1 - e^{-2x}$, $y = 2 \ln x$,
$y = 1 - \ln x$, $y = \ln(x+3)$.

Exponential and Log Functions — Solving Equations

Description. i) To solve an equation where unknowns appear in exponents, write both sides as a power of the same number if possible. Take the same log of both sides (equivalently equate exponents if both sides are a power of the same number). ii) To solve an equation with unknowns inside logarithm functions, express in terms of a single log. Convert to an exponential equation, and solve.

Examples. Solve i) $8^{(3x+1)} = 2^{(x-1)}$, i) $2^{(x+3)} = 97$,
ii) $\text{Log}(3x+1) - \text{Log}(x-2) = 1$. [Log denotes $\log_{10}$]

Solution. i) $8^{(3x+1)} = 2^{(x-1)}$
$(2^3)^{(3x+1)} = 2^{(x-1)}$
$2^{(9x+3)} = 2^{(x-1)}$
$9x + 3 = x - 1$
$8x = -4$
$x = -1/2$.

i) $2^{(x+3)} = 97$,
$\text{Log}[2^{(x+3)}] = \text{Log } 97$
$(x+3)\text{Log } 2 = \text{Log } 97$
$x + 3 = \text{Log } 97 / \text{Log } 2$
$x = -3 + \text{Log } 97 / \text{Log } 2$.

ii) $\text{Log}(3x+1) - \text{Log}(x-2) = 1$

$$\text{Log}\left(\frac{3x+1}{x-2}\right) = 1$$

$$\frac{3x+1}{x-2} = 10^1$$

$3x + 1 = 10x - 20$
$7x = 21$
$x = 3$.

Samples. i) Solve $5^{(x-4)} = 125$, $3^{(x+1)} = 4^{(x-1)}$
ii) Solve $\log_4 (x+6) + \log_4 (x-6) = 3$, $\log_x 16 = 2$, $\text{Log}(\text{Log } x) = 10$.

Exponential and Log Functions — Word problems

Description. Set up and solve problems involving exponential and logarithmic equations.

Example. Suppose that the growth function of a population is given by the exponential model $P(t) = 70\, e^{.005t}$ where $P(0) = 70$ gives the population (in millions) in 1986, and $P(t)$ gives the population t years after 1986. In what year will the population double from the 1986 level?

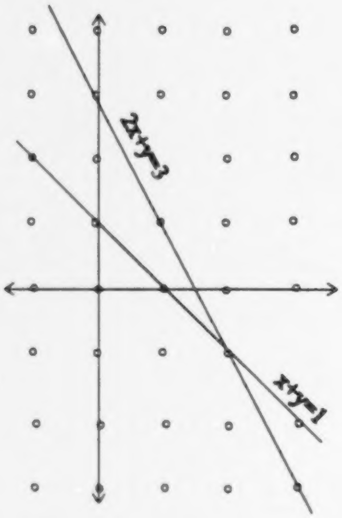
Solution. We first solve $P(t) = 140$ ($= 2\times 70$) or $70\, e^{.005t} = 140$ for t. This gives $t = (\ln 2)/.005$ which is approximately 140. The year in question is then $1986 + 140 = 2126$.

Systems of Equations/Inequalities, Matrices

Systems of Equations/Inequalities, Matrices — Graphical Solution

Description. Solve a system of equations by observing the intesection point(s) on the graphs of the equations.

Example. Solve by graphing
$2x + y = 3$
$x + y = 1$



Solution. $(x, y) = (2, -1)$.

Samples. Solve the following systems by graphing:
$x - y = 2$, $x + 3y = 10$,
$x = y^2$, $y = x$.

Systems of Equations/Inequalities, Matrices — Substitution method

Description. Solve a system by first solving one equation for one of the variables, and using the result to replace that variable in the other equation(s).

Example. Solve by substitution $2x + y = 3$, $x + y = 1$.

Solution. From the first equation $y = 3 - 2x$.
Substitute in the second equation to obtain $x + (3 - 2x) = 1$ or $x = 2$.
Hence $y = 3 - 2(2) = -1$, and the solution is $(x, y) = (2, -1)$.

Samples. Solve by substitution:
$x - y = 2$, $x + 3y = 10$,
$x^2 + 6x - y = 0$, $2x + y = -7$,
$y = x^3$, $y = x$.

Systems of Equations/Inequalities, Matrices — Elimination method

Description. Solve a system by first adding a multiple of one equation to the other, in order to eliminate a variable from the resulting equation. Identify "elementary operations", "triangular linear system", and "back substitution".

Examples. Solve by elimination

$$\begin{array}{rl} x + y = 3 & (1)\\ 2x + 5y = 9. & (2) \end{array}$$

Solution. Add -2 times equation (1) to equation (2) to obtain the triangular system

$$\begin{array}{l} x + y = 3\\ \quad 3y = 3. \end{array}$$

Hence $y = 1$. By back substitution equation (1) becomes $x + 1 = 3$. The solution is $(x, y) = (2, 1)$.

Sample. Solve by elimination

$$\begin{array}{l} 2x + 4y = 14\\ 3x - 5y = -1. \end{array}$$

Systems of Equations/Inequalities, Matrices — Linear systems with three variables

Description. Use elimination to solve a linear system with three of more variables.

Example. Solve by elimination

$$\begin{array}{rl} 4x + 5y - 3z = -6 & (E1)\\ -2x + y + 2z = 1 & (E2)\\ 2x + y - z = 0 & (E3). \end{array}$$

Solution. $(E_i) \leftrightarrow (E_j)$ indicates that equations (E_i) and (E_j) are interchanged. $c(E_i)$ indicates that equation (E_i) is multiplied by c. $c(E_i) +\!\!\longrightarrow (E_j)$ indicates that c times equation (E_i) is added to equation (E_j).

$$\begin{array}{rlcrl} 4x + 5y - 3z = -6 & (E1) & & 2x + y - z = 0 & (E1)\\ -2x + y + 2z = 1 & (E2) & (E1)\leftrightarrow(E3) & -2x + y + 2z = 1 & (E2)\\ 2x + y - z = 0 & (E3). & & 4x + 5y - 3z = -6 & (E3) \end{array}$$

$$\begin{array}{lrlc} 1(E1)+\!\!\longrightarrow(E2) & 2x + y - z = 0 & (E1) & \\ -2(E1)+\!\!\longrightarrow(E3) & 2y + z = 1 & (E2) & 2(E3)\\ & 3y - z = -6 & (E3) & \end{array}$$

$$\begin{array}{rlcrl} 2x + y - z = 0 & (E1). & -3(E2)+\!\!\longrightarrow(E3) & 2x + y - z = 0 & (E1)\\ 2y + z = 1 & (E2) & \text{gives the} & 2y + z = 1 & (E2)\\ 6y - 2z = -12 & (E3) & \text{triangular form} & -5z = -15 & (E3) \end{array}$$

Hence, $z = 3$. By back substitution in (E2) $2y + 3 = 1$ or $y = -1$. Back substitute in (E1) to obtain $2x - 1 - 3 = 0$ or $x = 2$. The solution is $(x, y, z) = (2, -1, 3)$.

Systems of Equations/Inequalities, Matrices — Word Problems

Description. Set up and solve word problems involving systems of equations.

Example. Bob noticed the following about his monthly paycheck. The total monthly salary of $3200 consists of withholding for taxes, withholding for benefits, and take home pay. The total of the two withholding items is $800 less than his take home pay, and the difference between the tax withholding and the benifits withholding is $400. How much is withheld for taxes and benefits, and what is Bob's take home pay?

Solution. Let x = tax withholding, y = benefit withholding, and z = take home pay. The information given can then be written as $x + y + z = 3200$, $x + y = z - 800$, and $x - y = 400$. which yields the system of equations

$$\begin{aligned} x+y+z &= 3200 \\ x+y-z &= -800. \\ x-y \quad &= \ 400 \end{aligned}$$

The solution of this system gives x = $800 tax withholding, y = $400 benefits with holding, and z = $2000 take home pay.

Systems of Equations/Inequalities, Matrices — Inconsistent and dependent systems

Description. i) Use elimination to recognize inconsistent linear systems. ii) Use elimination to specify solutions of consistent dependent linear systems, giving multiple solutions as appropriate.

Example. i) Solve

$$\begin{aligned} x - 2y &= 3 \quad (E1) \\ -4x + 8y &= 0 \quad (E2). \end{aligned}$$

ii) Solve

$$\begin{aligned} x - y + z &= 2 \quad (E1) \\ 3x - 4y + 2z &= 1 \quad (E2) \\ 4x - 5y + 3z &= 3 \quad (E3). \end{aligned}$$

Solution. i) 4(E1)+→(E2) gives $0 = 12$ for (E2). This is inconsistent, and the original system has no solution.

ii)

$$\begin{array}{lll} x - y + z = 2 \quad (E1) & -3(E1)+\rightarrow(E2) & x - y + z = 2 \quad (E1) \\ 3x - 4y + 2z = 1 \quad (E2) & -4(E1)+\rightarrow(E3) & -y - z = -5 \quad (E2) \\ 4x - 5y + 3z = 3 \quad (E3). & & -y - z = -5 \quad (E3). \end{array}$$

Since (E2) = (E3) the system is dependent, and we can drop (E3). Solving (E1) and (E2) for x and y respectively in terms of z and back substituting for y in (E1) yields $x = 7 - 2z$ and $y = 5 - z$. All solutions of the system are thus given by $(x, y, z) = (7 - 2t, 5 - t, t)$ where t can be any real number.

Samples. Solve any of the following systems which are consistent. When possible, give multiple solutions.

$$\begin{matrix} x+2y=3 \\ 4x+8y=12 \end{matrix}, \quad \begin{matrix} x-y=1 \\ 2x-2y=0 \end{matrix}, \quad \begin{matrix} 3x+y-z=0 \\ x-y+2z=1 \\ 4x+z=1 \end{matrix}.$$

Systems of Equations/Inequalities, Matrices — Elimination using matrices

Description. Transform a system of linear equations to its augmented matrix, reduce to triangular form with elementary row operations, and solve the resulting equivalent system by back substitution.

Example. Using elimination on the augmented matrix, solve

$$\begin{aligned} 4x + 5y - 3z &= -6 \\ -2x + y + 2z &= 1 \\ 2x + y - z &= 0. \end{aligned}$$

Solution. Using a similar notation for elementary row operations (E for equation is replaced by R for row):

$$\left(\begin{array}{ccc|c} 4 & 5 & -3 & -6 \\ -2 & 1 & 2 & 1 \\ 2 & 1 & -1 & 0 \end{array}\right) \quad (R1)\leftrightarrow(R3) \quad \left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ -2 & 1 & 2 & 1 \\ 4 & 5 & -3 & -6 \end{array}\right)$$

$$\begin{matrix} 1(R1)+\rightarrow(R2) \\ -2(R1)+\rightarrow(R3) \end{matrix} \quad \left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 3 & -1 & -6 \end{array}\right) \quad 2(R3)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 6 & -2 & -12 \end{array}\right) \quad -3(R2)+\rightarrow(R3) \quad \left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & -5 & -15 \end{array}\right)$$

This gives the triangular system

$$\begin{aligned} 2x + y - z &= 0 \\ 2y + z &= 1 \\ -5z &= -15. \end{aligned}$$

Using back substitution we obtain the solution $(x, y, z) = (2, -1, 3)$.

Samples. Use elimination on the augmented matrices to solve:

$$\begin{matrix} x-y+z=2 \\ 3x-2y+z=5 \\ 2x-y-z=2 \end{matrix}, \quad \begin{matrix} 2x-y+z=0 \\ 6x-4y+2z=0 \\ 8x-5y+3z=0 \end{matrix}, \quad \begin{matrix} w+x+y+z=2 \\ w+2x+2y+4z=1 \\ -w+x-y-z=-6 \\ -w+3x+y-z=-2 \end{matrix}.$$

Systems of Equations/Inequalities, Matrices — Arithmetic operations on matrices

Description. Add, subtract, scalar multiply, and multiply conformal matrices.

Example. Given: $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $H = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$.
Find $A + B$, $B - A$, AB, AC, AX, $2A$, and BH.

Solution. $A + B = \begin{pmatrix} 4 & 3 \\ 4 & 4 \end{pmatrix}$, $B - A = \begin{pmatrix} 2 & -1 \\ 0 & -2 \end{pmatrix}$, $AB = \begin{pmatrix} 7 & 3 \\ 12 & 5 \end{pmatrix}$,

$AC = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \end{pmatrix}$, $AX = \begin{pmatrix} x + 2y \\ 2x + 3y \end{pmatrix}$, $2A = \begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix}$ $BH = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$.

Samples. For the matrices given in the example, find: $X + H$, $A - B$, $-3C$, BX, AH, BA, $AX + H$, and $A(X - H)$.

Systems of Equations/Inequalities, Matrices — Linear systems as AX = b

Description. Write a linear system of equations in the matrix form $AX = b$, where A is called the coefficient matrix of the system.

Example. Write the system $\begin{matrix} x - y = 2 \\ 3x + 4y = 5 \end{matrix}$ in matrix form.

Solution. $\begin{pmatrix} 1 & -1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

Samples. Write the following systems in matrix form:

$$\begin{matrix} x + y = 3 \\ 2x + 3y + 4z = 5 \\ 3x - 5y + z = 2, \end{matrix} \qquad \begin{matrix} 2x - 3y + z = 1 \\ 3x + 7z = 3, \end{matrix} \qquad \begin{matrix} -x + 2y = 3 \\ 2x - 5y = -2 \\ x + 6y = 7 \end{matrix}$$

Systems of Equations/Inequalities, Matrices Inverses of matrices

Description. i) Use the Gauss-Jordan method of finding the multiplicative inverse of an invertible matrix. ii) For a system of linear equations with an invertible coefficient matrix, use the inverse of the coefficient matrix to solve the system.

Example. i) Find the inverse of $\begin{pmatrix} 2 & 2 \\ 4 & 3 \end{pmatrix}$, and

ii) use the inverse to solve $\begin{matrix} 2x + 2y = 6 \\ 4x + 3y = 11 \end{matrix}$.

Solution. i) $\left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right)$

$\to \left(\begin{array}{cc|cc} 2 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -1.5 & 1 \\ 0 & 1 & 2 & -1 \end{array}\right)$

$\begin{pmatrix} 2 & 2 \\ 4 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -1.5 & 1 \\ 2 & -1 \end{pmatrix}$.

ii) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1.5 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 6 \\ 11 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Samples. i) Find the inverse of $\begin{pmatrix} 1 & 4 & -1 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{pmatrix}$.

ii) Use the inverse of the coefficient matrix to solve $\begin{matrix} -2x + 2y = 1 \\ 3x + y = 4 \end{matrix}$

Description. (i) Evaluate 2-by-2 determinants by the formula $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, (ii) evaluate 3-by-3 determinants by expansion or reduction, and (iii) use Cramer's rule to solve linear systems of two equations in two unknowns.

Example. (i) Evaluate the determinant $\begin{vmatrix} 2 & 1 & 4 \\ 0 & 3 & 7 \\ 2 & 1 & 9 \end{vmatrix}$.

(ii) Solve the system $\begin{matrix} 2x + 3y = 4 \\ 3x + 4y = 7 \end{matrix}$ using Cramer's rule.

Solution. (i) Expanding by the first column:

$$\begin{vmatrix} 2 & 1 & 4 \\ 0 & 3 & 7 \\ 2 & 1 & 9 \end{vmatrix} = 2\begin{vmatrix} 3 & 7 \\ 1 & 9 \end{vmatrix} - 0\begin{vmatrix} 1 & 4 \\ 1 & 9 \end{vmatrix} + 2\begin{vmatrix} 1 & 4 \\ 3 & 7 \end{vmatrix} = 2(27 - 7) + 2(7 - 12) = 30.$$

Alternatively, one could reduce the determinant to triangular form by subtracting the first row from the last row:

$$\begin{vmatrix} 2 & 1 & 4 \\ 0 & 3 & 7 \\ 2 & 1 & 9 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 4 \\ 0 & 3 & 7 \\ 0 & 0 & 5 \end{vmatrix} = 2 \times 3 \times 5 = 30.$$

(ii) $x = \dfrac{\begin{vmatrix} 4 & 3 \\ 7 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}} = \dfrac{-5}{-1} = 5$, and $y = \dfrac{\begin{vmatrix} 2 & 4 \\ 3 & 7 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}} = \dfrac{2}{-1} = -2$.

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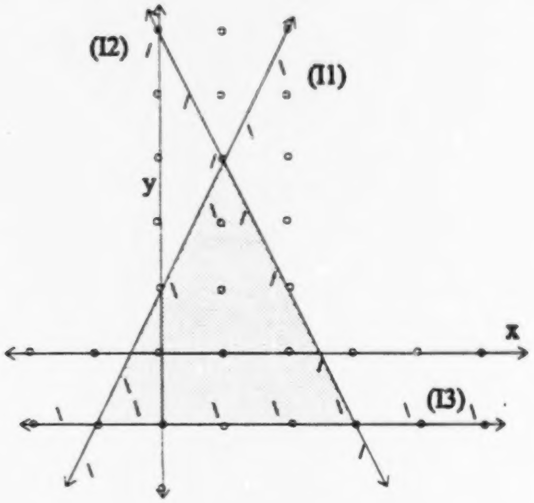
Description. Graph the solution set of a system of inequalities by intersecting the graphs of the solution set of each inequality in the system.

Example. Graph the solution set of

$$\begin{aligned} -2x+y &\le 1, & \text{(I1)} \\ 2x+y &\le 5, & \text{(I2)} \\ -y &\le 1, & \text{(I3)} \end{aligned}$$

Solution.



Samples. Graph the solution sets of the following systems of inequalities:

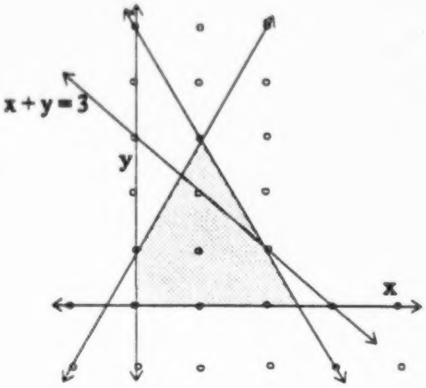
$$\begin{gathered} 2x+y \le 16 \\ x \ge 0 \\ y \ge 0 \end{gathered} \qquad \begin{gathered} y-4x \le 8 \\ -1 \le x \le 2 \\ y \ge x \end{gathered} \qquad \begin{gathered} y \ge x^2 \\ y-2x \le 3 \end{gathered}$$

Systems of Equations/Inequalities, Matrices Linear programming

Description. Solve linear programming problems in two variables by graphing.

Example. Find the maximum value of $x + y$ subject to the constraints: $-2x + y \leq 1$, $2x + y \leq 5$, $-y \leq 1$, and $x \geq 0$, $y \geq 0$.

Solution. Using the results of the previous example, the feasible values of (x, y) are given by the shaded region. Now any line $x + y = c$ is parallel to the plotted line $x + y = 3$. The line $x + y = c$, with the largest value of c which intersects the shaded region, will intersect the region at the point $(x, y) = (1, 4)$. The largest value of $x + y$ for this region is then $c = 1 + 4 = 5$.



Systems of Equations/Inequalities, Matrices Nonlinear systems

Description. Solve nonlinear systems by substitution.

Example. Find the points of intersection of the line $x + y = 5$ with the parabola $y = 1 + 4x - x^2$.

Solution. Substituting $1 + 4x - x^2$ for y in $x + y = 5$ gives $x + 1 + 4x - x^2 = 5$ or $0 = x^2 - 5x + 4$. By factoring, $x = 1$ or 4. Corresponding y values are then determined from $y = 5 - x$, giving the points of intersection $(1, 4)$, and $(4, 1)$.

Polynomials

Polynomials — Graphs of polynomial functions

Description. i) Use continuity, x-intercepts, and test values between intercepts (or rules for multiplying signed numbers) to sketch the graph of a polynomial function that is the product of distinct linear factors. ii) Use test values and rule for odd and even exponents to sketch the graphs of polynomial functions of the form $y = ax^n$. iii) Have some general understanding of how the graph of a general polynomial function may be similar to or differ from the above cases.

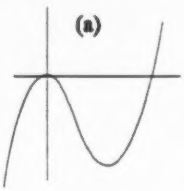
Examples.

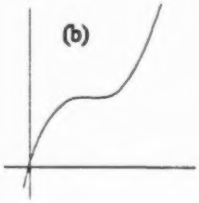
i) Sketch the graph of $f(x) = x^3 - 4x$.

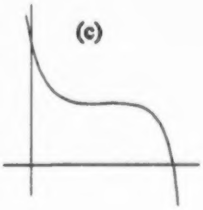
ii) Sketch the graph of $f(x) = -x^3$.

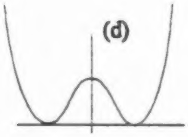
iii) Match the following polynomials to their graphs:

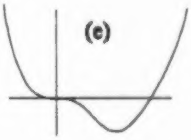
$f(x) = (x - 1)^3 + 1$, $g(x) = x^3 - x^2$, $h(x) = x^4 - x^3$,
$j(x) = x^4 - 2x^2 + 1$ $k(x) = 1 - (x-1)^5$.





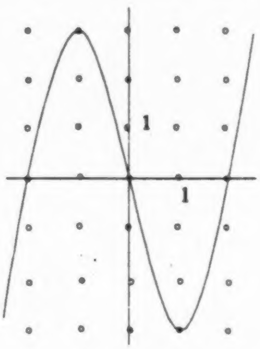




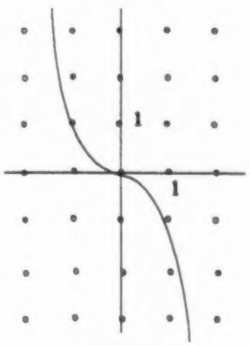


Solutions.

i) $f(x) = x^3 - 4x$ is a cubic with positive leading coefficient, and three distinct roots -1, 0, 1.



ii) $f(x) = -x^3$ has a negative leading coefficient, and a triple root at 0.



iii) $f(x) \leftrightarrow (b)$ may be obtained from $y = x^3$ by a shift of axes.

$g(x) \leftrightarrow (a)$ is a cubic with positive leading coefficient, a double root at 0 and a simple root at 1.

$h(x) \leftrightarrow (e)$ is a quartic with positive leading coefficient, a triple root at 0, and a simple root at 1.

$j(x) \leftrightarrow (d)$ is a quartic with positive leading coefficient, a double root at -1, and a double root at 1.

$k(x) \leftrightarrow (c)$ may be obtained by a shift of axes from $y = -x^5$.

Samples. i) Sketch the graphs of $f(x) = x^3 - x$, $g(x) = x^3 - 4x^3 + 3x$, $h(x) = x^4 - 5x^2 + 4$, $k(x) = x\,h(x)$.

ii) Sketch the graphs of $f(x) = x(x + 1)^2$ $g(x) = x(x - 1)^2$
$h(x) = x(x + 1)^2 + 1$ $j(x) = -(x - 2)^3$ $k(x) = x^2(x^2 - 1)$
$p(x) = x(x - 1)^2(x + 1)$ $q(x) = x^3(x - 1)$ $r(x) = x^3(x + 1)$
$s(x) = x(x - 1)(x + 1)^2$

Polynomials — Division of polynomials and the division algorithm

Description. i) Use long division to divide one polynomial by another. ii) Interpret the results of long division in the form dividend = (divisor)(quotient) + remainder where the remainder is 0 or has degree less than the degree of the divisor.

Example. i) Divide $x^4 + x^3 + 5x^2 + 4x + 3$ by $x^2 + 2$, and ii) interpret the results.

Solution. i)

$$
\begin{array}{r|l}
 & \qquad\qquad\quad x^2+x+3 \\ \hline
x^2+2 & x^4+x^3+5x^2+4x+3 \\
 & \underline{x^4 \qquad\;\; +2x^2} \\
 & \qquad x^3+3x^2+4x+3 \\
 & \qquad \underline{x^3 \qquad\;\; +2x} \\
 & \qquad\qquad 3x^2+2x+3 \\
 & \qquad\qquad \underline{3x^2 \qquad\; +6} \\
 & \qquad\qquad\qquad 2x-6
\end{array}
$$

ii) $x^4 + x^3 + 5x^2 + 4x + 3 = (x^2 + 2)(x^2 + x + 3) + (2x - 6)$.

Polynomials Roots and factors

Description. i) Evaluate a polynomial p(x) at several numerical values of x to determine whether these values are roots (zeros) of the polynomial. ii) By observing whether the remainder in a long division is 0 (or using other factoring techniques), determine whether one polynomial is a factor of another. If so, determine the other corresponding factor.

Example. i) Determine whether -1, 1, and 2 are roots of $p(x) = 2x^3 - 3x^2 + 2x - 1$.
ii) Determine whether $x^2 + 1$ and $x^2 + 6$ are factor of $q(x) = x^4 + 8x^2 + 12$.

Solution. i) $p(-1) = -8$, $p(1) = 0$, $p(2) = 7$, so that of the values considered, only 1 is a root of p(x).
ii) q(x) divided by $x^2 + 1$ gives a remainder of 5, so that $x^2 + 1$ is not a factor of q(x). q(x) divided by $x^2 + 6$ gives a remainder of 0 and a quotient of $x^2 + 2$, so that $x^2 + 6$ and $x^2 + 2$ are factors of q(x). This last result can also be obtained by quadratic factoring techniques, since q(x) is a quadratic in x^2.

Polynomials Synthetic division

Description. Use synthetic division to divide a polynomial by a polynomial of the form x - r. Also determine the quotient and remainder.

Example Use synthetic division to divide $x^5 - 3x^3 + 2x^2 + x - 1$ by $x + 2$.

Solution.

$$\begin{array}{r|rrrrrr} \underline{-2} & 1 & 0 & -3 & 2 & 1 & -1 \\ & & -2 & 4 & -2 & 0 & -2 \\ \hline & 1 & -2 & 1 & 0 & 1 & -3 \end{array}$$

gives a quotient of $x^4 - 2x^3 + x^2 + 1$ and a remainder of -3, that is
$x^5 - 3x^3 + 2x^2 + x - 1 = (x^4 - 2x^3 + x^2 + 1)(x + 2) - 3$.

Polynomials Remainder theorem

Description. Use the remainder from dividing p(x) by x - r to give the polynomial function value p(r).

Example. Use division to find p(2) where $p(x) = -x^3 + x^2 + 4x + 1$.

Solution. p(x) divided by x - 2 has remainder p(2) = 5.

Polynomials Fundamental theorem of algebra

Description. Use the following theorem to know how many roots to find when listing the roots of a polynomial. **Fundamental Theorem of Algebra**: Every polynomial of degree n, $n > 0$, with real or complex coefficients, can be factored into n linear factors over the complex number field.

Example. Find the roots of $p(x) = x^7 + 14x^5 - 32x^3$.

Solution. $p(x) = x^3(x^4 + 14x^2 - 32) = (x - 0)^3(x^2 - 2)(x^2 + 16)$
$= (x - 0)^3(x - \sqrt{2})(x + \sqrt{2})(x - 4i)(x + 4i)$
so the 7 roots of p(x) (counting multiplicities) are: 0, 0, 0, $\sqrt{2}$, $-\sqrt{2}$, 4i, -4i.

Polynomials Roots occuring in pairs

Description. Use the following theorem as an aid in factoring polynomials.
Theorem. a) If p(x) is a polynomial with rational coefficients and a root $a + \sqrt{b}$, where a and b are rational and $\sqrt{b}$ is irrational, then $a - \sqrt{b}$ is also a root of p(x) so that $[x - (a + \sqrt{b})][x - (a - \sqrt{b})] = x^2 - 2ax + a^2 - b$ is a factor of p(x).
b) If p(x) is a polynomial with real coefficients and a root $a + bi$, where a and b are real, then $a - bi$ is also a root of p(x) so that $[x - (a + bi)][x - (a - bi)] = x^2 - 2ax + a^2 + b^2$ is a factor of p(x).

Example. Given that i is a root of $p(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$, find a complete factorization of p(x) over the over the real number field.

Solution. Since i is a root of p(x), so is -i, and $(x - i)(x + i) = x^2 + 1$ is a factor of p(x). Dividing p(x) by $x^2 + 1$ gives $p(x) = (x^2 + 1)(x^2 - 5x + 6)$. Then a complete factorization of p(x) over the real number field is $p(x) = (x^2 + 1)(x - 2)(x - 3)$.
[The roots of p(x) are i, -i, 2, and 3.]

Polynomials Polynomials with given roots

Description. Given the roots of a polynomial p(x), multiply out the corresponding linear factors to find p(x).

Example. Find the polynomial of smallest degree with real coefficients, leading coefficient 1, and roots 3i and 1.

Solution. Since the coefficients are real and 3i is a root, then -3i is also a root. Hence $p(x) = (x - 3i)(x + 3i)(x - 1) = x^3 - x^2 + 9x - 9$.

Algebra Inventory -- Summary of Topics

Real Numbers	
Kinds (positive/negative, rational/irrational)	
Arith. of signed numbers, additive inverses	
Properties of + and ×	
Order	
Scientific notation	
Equivalent expressions	
Order of operations	
Integer exponents Notation	______
Laws	______
Absolute value (definition and properties)	
Intervals	
Union and intesection of sets	
Arithmetic of complex numbers	

Algebraic Expressions (AEs)	
Polynomials (definition, recognition)	
Addition and subtraction of AEs	
Multiplying polynomials	
Factoring polynomials - common factor	
Factoring polynomials - difference of squares	
Factoring quadratics	
Factoring by quadratic technique	
Factoring a sum or difference of cubes	
Fractional expressions - multplying and dividing	
Fractional expressions - simplifying	
Fractional expressions - adding and subtracting	
Fractions of fractions - simplifying	
Radical notation	
Rational exponents, definition, laws	

Equations and Inequalities in One Unknown	
General principles	
Linear equations	
Quadratic equations - factoring	
Quadratic equations - completing the square	
Quadratic formula - real roots	____
- discriminant	____
- complex roots	____
Equations with radicals	
Equations reducible to quadratics	
Equations with fractional expressions	
Linear inequalities	
Equations with absolute value	
Inequalities with absolute value	
Quadratic inequalities	
Inequalities with fractional expressions	

Problem Solving, Word Problems	
General techniques	
Ratio and Proportion	
Percent	
Interest simple	____
compound	____
Travel (speed, velocity)	
Geometric formula (Area, etc.)	
Geometry of lines and similar triangles	
Mixtures	
Averages, means	
Units of measurement	
Variation	

Functions	
Cartesian product (Ordered pairs)	
Relations, domain, range	
Recognizing functions: domain, range	
Graphing relations and functions	
Vertical line test	
Shift of axes, Vertical shift -- f(x) + k Horizontal shift -- f(x+h)	____ ____
Notation for functions, Function values f(x) Difference quotient	____ ____
Algebra of functions	
Composition	
Increasing/Decreasing functions	
Inverses of relations and functions	
Symmetry about y = x	
One to One functions, horizontal line test	
Formula for an inverse	
Even and Odd functions	

Equations and Inequalities in Two Variables	
Linear equations - graphing, slope	____ ____
Lines - Point slope equation	
Lines - Point intercept equation	
Parallel and perpendicular lines	
Distance formula in the plane	
Midpoints of line seqments	
Equations of circles	
Parabolas - graphing parabolas - intercepts - vertex, max/min - symmetry	____ ____ ____ ____
Quadratic word problems	

Exponential and Log Functions	
Graphs of exponential functions including shifts of axes	______ ______
Log functions as inverses of exponential functions	
Graphs of log functions including shifts of axes	______ ______
Domain and range of exp. and log functions	
Conversion: Log ↔ Exp	
Properties of logs	
Change of base	
Natural logs	
Solving equations	
Word problems	

Systems of Equations/Inequalities, Matrices	
Graphical solution	
Substitution method	
Elimination/Addition method	
Linear systems with three variables	
Word problems	
Inconsistent systems, and Dependent systems	______ ______
Elimination using matrices, augmented matrix reduced form	______ ______
Arithmetic operations on matrices	
Linear system as AX = b	
Inverses of matrices	
Determinants, and Cramers rule	______ ______
Systems of inequalities	
Linear programming	
Nonlinear systems	

Polynomials	
Graphs of polynomial functions	
Division algorithm	
Roots and factors	
Remainder theorem	
Synthetic division	
Fundamental theorem of algebra	
Polynomials with given roots	

Appendix F

National Center for Academic Achievement & Transfer Project

Luncheon Workshop

F-1

NATIONAL CENTER FOR ACADEMIC ACHIEVEMENT & TRANSFER PROJECT

Luncheon Workshop

Georgia State University and Atlanta Metropolitan College Partnership Project
Center for the Study of Adult Literacy

◆◆◆

New Georgia Railroad Depot Blue Room
11:30 a.m. to 2:00 p.m.
February 16, 1993

◆◆◆

BUFFET LUNCHEON	
WELCOME & INTRODUCTIONS	**Joanne R. Nurss**, *Director* *Center for the Study of Adult Literacy, GSU*
GSU/AMC FACULTY PANEL	*Georgia State University/Atlanta Metropolitan College Partnership Project*
Overview of the Project	**Lynn Hart** *Division of Developmental Studies, GSU*
Content Analysis	**John Morrell** *Division of Natural Sciences & Mathematics, AMC*
Observation Instrument	**Deborah Najee-ullah** *Division of Developmental Studies, GSU*
Algebra Inventory	**Frank Hall** *Department of Mathematics & Computer Science, GSU*
AMC Faculty Workshop	**Barbara Morgan**, *Chair* *Division of Natural Sciences & Mathematics, AMC*
INTRODUCTION OF SPEAKER	**Barbara Morgan**, *Chair* *Division of Natural Sciences & Mathematics, AMC*
SPEAKER	**William Haver**, *Professor of Mathematical Sciences* *Virginia Commonwealth University, Richmond* *"Collaborative Teaching at J. Sargeant Reynolds Community College and Virginia Commonwealth University"*
QUESTIONS AND DISCUSSION	
CONCLUSION	

Appendix G

GSU-AMC Math Project Curriculum Revision Workshop

GSU-AMC Math Project
Curriculum Revision Workshop

The GSU-AMU Math Project provided the impetus for a curriculum revision workshop for Atlanta Metropolitan College Faculty and Administrators. The workshop was held December 10th and 11th (1992) at the Peachtree Executive Conference Center, Peachtree City, Georgia. The presenter for the workshop sessions was Dr. Harvey Brightman, Regents' Professor of Decision Sciences at Georgia State University. Dr. Brightman's presentations (see attached agenda) were focused on improving instruction at Atlanta Metropolitan College with specific emphasis on writing objectives, implementing instructional strategies and evaluating instruction for different levels of learning (especially the higher levels related to critical thinking). Workshop goals were to:

1. provide a theoretical framework for the review and enhancement of course syllabi.

2. initiate the revision of course objectives to reflect institutional priorities which include:
   a. improving student performance/scores on area exams.
   b. facilitating the transfer of AMC students to senior institutions.
   c. improving the critical thinking skills/abilities of Atlanta Metropolitan students.

Funds made available through this partnership grant from the Foundation administered by the American Council on Education supported fourteen fulltime and part-time faculty members from the Natural Science and Mathematics Division and the College's Developmental Studies Division. Grant funds were also used to provide partial compensation for the presenter, Dr. Brightman.

Mathematics faculty members made very positive comments in responding to an evaluation of workshop content and format. In fact, information gleaned from (1) the workshop, (2) the algebra inventory, and (3) dialogue with math faculty at GSU is currently being used by the Math Faculty at AMC to revise the Math 101 (college algebra) syllabus. Dr. Zacharia Manare, a GSU-AMC Math Project Participant, is leading the math faculty in this curriculum revision initiative. Two other members of the curriculum revision committee were also directly involved with the grant.

TENTATIVE AGENDA

December 10th

Day 1

11:30-12:00	Arrival*
12:00- 1:00	Lunch
1:00- 4:15	1st General Session Writing Objectives For Different Levels of Learning
4:15- 4:30	Break
4:30- 6:30	2nd General Session Classroom Strategies For Achieving Different Levels of Learning
6:30-7:30	Dinner
7:30- until	R & R

December 11

Day 2

7:30-8:30	Breakfast
8:30-9:15	(Carry Over) Classroom Strategies For Achieving Different Levels of Learning
9:15-9:30	Break
9:30-12:00	3rd General Session Testing For Different Levels of Learning

*Check-in time is 3:00 p.m.; however, you may be able to check-in prior to 3:00 p.m. Upon arrival, bags may be checked at the concierge station.

Appendix H

Dissemination Efforts of the GSU-AMC Partnership Project

Publications

Bevis, J.H. & Hall, F.J. (1993). College Algebra Inventory. Atlanta, GA: Center for the Study of Adult Literacy, Georgia State University.

Conferences

Georgia State University and Atlanta Metropolitan College Partnership Project Workshop, New Georgia Railroad Depot, Atlanta, Georgia, February 16, 1993.

Workshops & Speeches

"The Atlanta Metropolitan College - Georgia State University Partnership Project," presented at the annual meeting of the Georgia Association of Developmental Educators, Jekyll Island, Georgia, April 26, 1993, by Lynn Hart & Deborah Najee-ullah.

"The Atlanta Metropolitan College - Georgia State University Partnership Project," presented at the annual Mathematics Conference, DeKalb College, Georgia, May 14, 1993, by Lynn Hart, Deborah Najee-ullah, Jean Bevis, & Frank Hall.